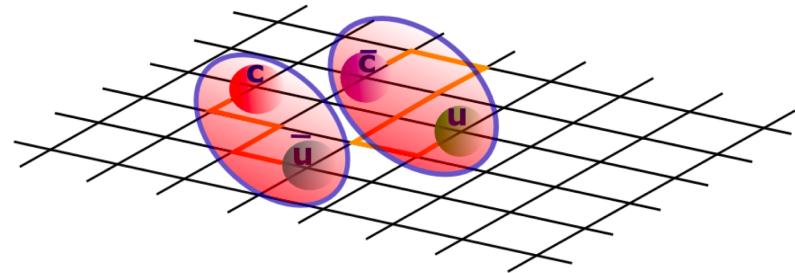


Charmonium(like) and other near-threshold mesons on the lattice



Sasa Prelovsek

University of Ljubljana & Jozef Stefan Institute, Slovenia

Jefferson Lab, Virginia, USA

Fermilab, 31st July 2015

Outline

- Experimental appetizer concerning exotic hadrons:
 - very recent discovery of pentaquarks
 - recent discoveries of tetraquarks (just one example)
- Theoretical approach to study
conventional and exotic hadrons
based directly on QCD – lattice QCD

Hadrons

Conventional



Normal baryon



Normal meson

x

Exotic

minimal valence content

$\bar{q} \bar{q} q q$

tetraquarks

$\bar{q} q q q q$

pentaquarks

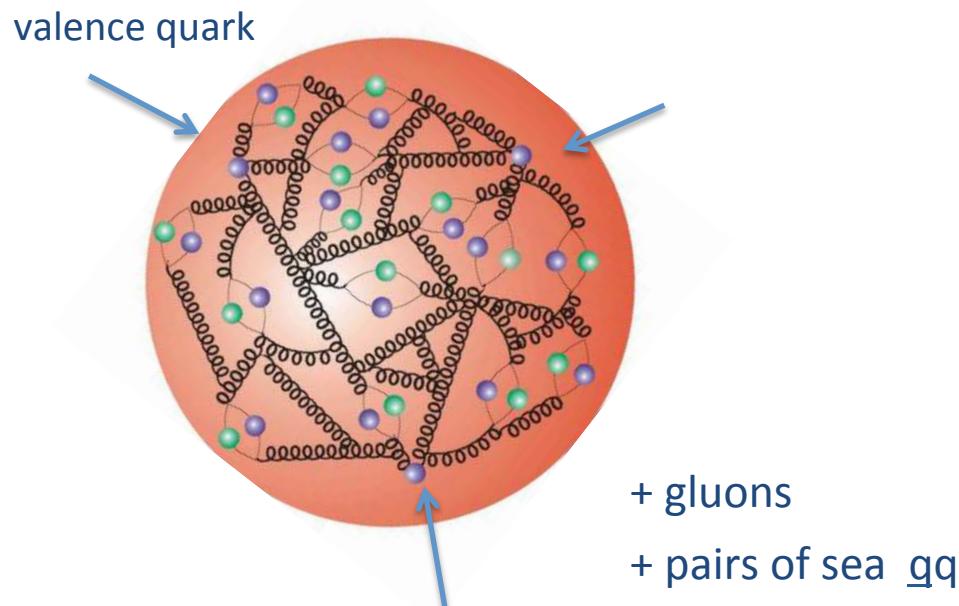
....

hybrids

glueballs

....

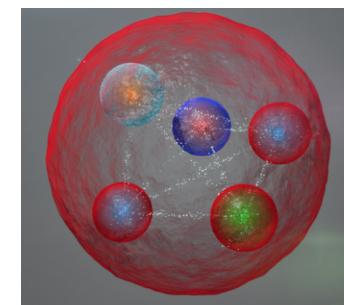
Terminology in this talk: tetra(penta) quarks indicate just the number of valence quarks in the state; it is not meant to say anything on how quarks are clustered in them



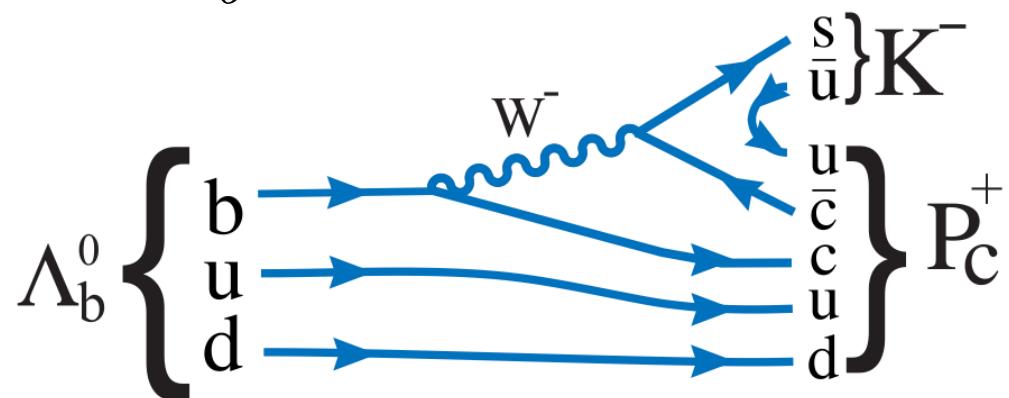
Pentaquarks

LHCb: 1507.03414

14th July 2015

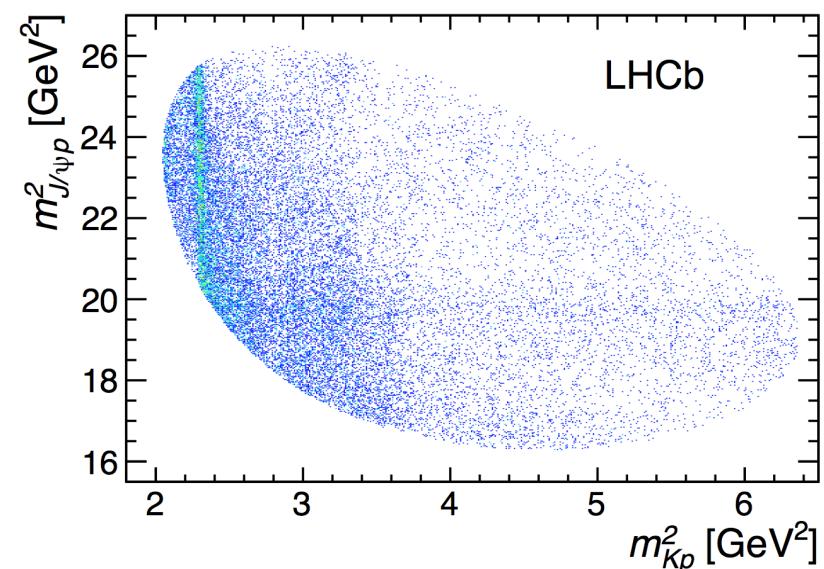
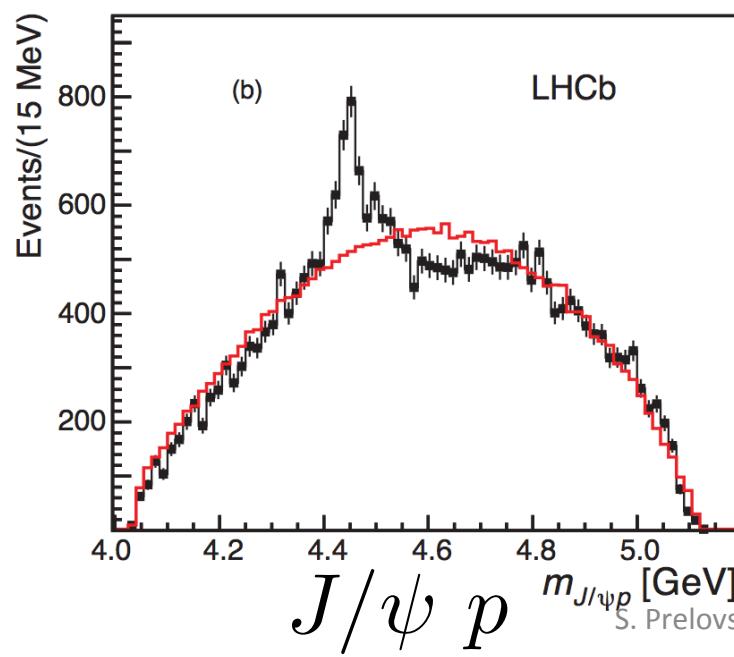


$$\Lambda_b^- \rightarrow K^- J/\psi p$$

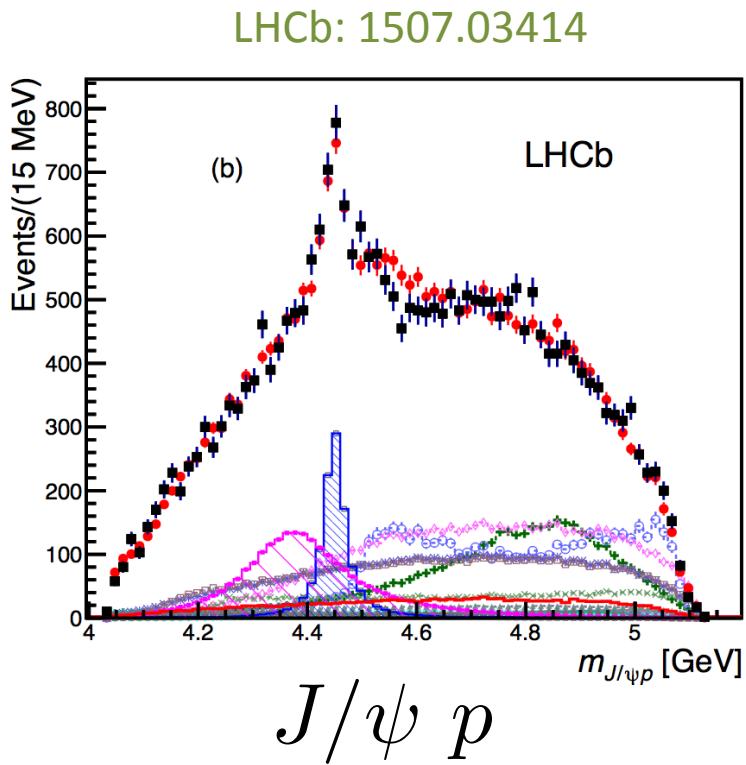


$$P_c^+ \rightarrow J/\psi \ p$$

$$\bar{c}c \quad uud$$



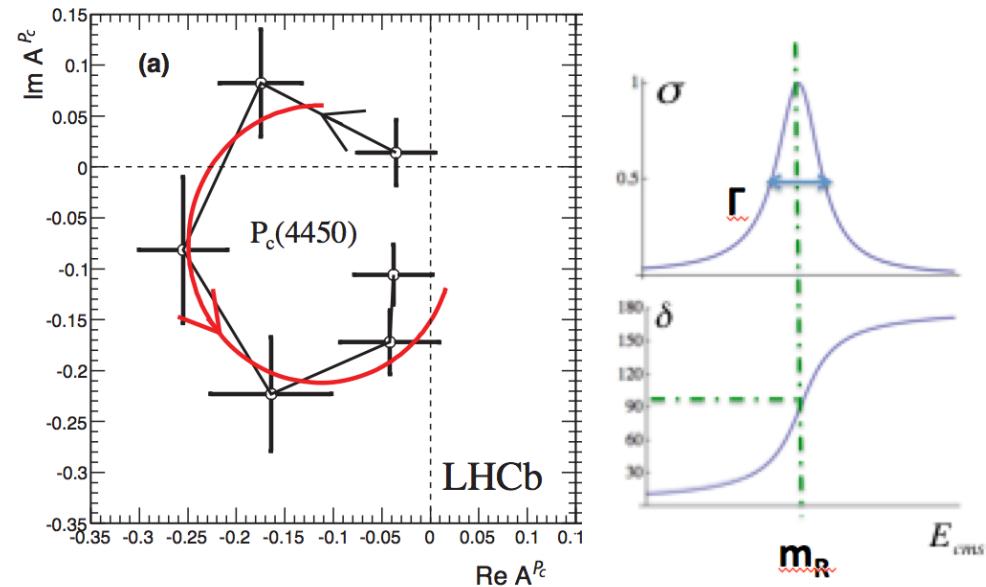
Two pentaquark candidates from LHCb



scattering matrix T for elastic scattering

$$A \simeq T = \frac{1}{2i}(e^{2i\delta(E)} - 1)$$

$$|T + \frac{1}{2i}| = \left| \frac{1}{2i} e^{2i\delta(E)} \right| = \frac{1}{2}$$



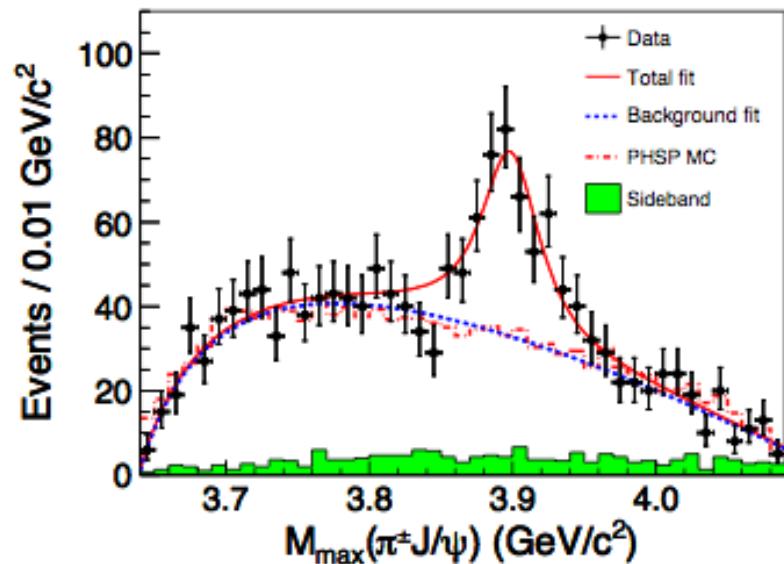
$P_c(4380)$: $M_1 = 4380 \pm 8 \pm 29 \text{ MeV}$ $\Gamma_1 = 205 \pm 18 \pm 86 \text{ MeV}$ $J^P = 3/2^+ \text{ or } 5/2^+$

$P_c(4450)$: $M_2 = 4449.8 \pm 1.7 \pm 2.5 \text{ MeV}$ $\Gamma_2 = 39 \pm 5 \pm 19 \text{ MeV}$ $J^P = 5/2^- \text{ or } 3/2^-$

Tetraquarks Z_c

BESIII, Belle, Cleo-c 2013

Example: $Z_c^+(3900)$



DD* thr.
↓

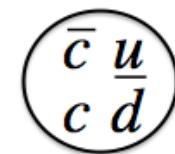
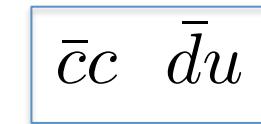
[BESIII, 2013, 1303.5949, PRL]

state confirmed by Belle, Cleo-c

$$Z_c^+(3900) \rightarrow J/\psi \pi^+$$

flavor content

of the hadron:



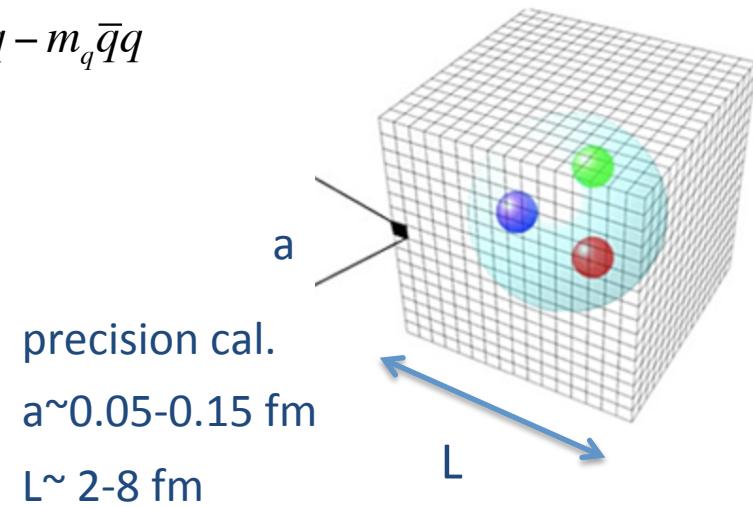
$$M = 3888 \pm 3 \text{ MeV}, \quad \Gamma = 35 \pm 7 \text{ MeV}$$

Non-perturbative method: QCD on lattice

$$L_{QCD} = -\frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} + \sum_{q=u,d,s,c,b,t} \bar{q} i \gamma_\mu (\partial^\mu + ig_s G_a^\mu T^a) q - m_q \bar{q} q$$

input: g_s , m_q

output: hadron properties
hadron interactions

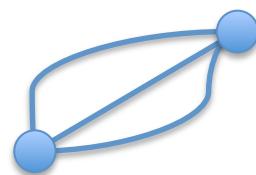


Evaluation of Feynman path integrals in discretized space-time

quantum mechanics

$$\int Dx e^{i S/\hbar}$$

$$S = \int dt L[x(t)]$$



quantum field theory in Euclidian space-time

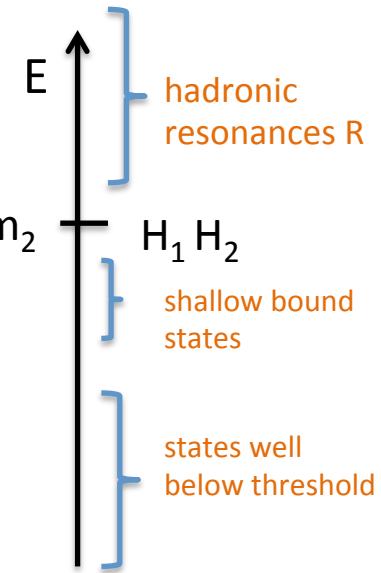
$$\int DG Dq D\bar{q} e^{-S_{QCD}/\hbar}$$

$$S_{QCD} = \int d^4x L_{QCD}[G(x), q(x), \bar{q}(x)]$$

x, t (Minkovsky) \rightarrow $x, i t$ (Euclidean)

Status of hadron spectroscopy on lattice

strong decay threshold: $m_1 + m_2$



The location of the threshold plays a major role for each channel J^{PC} .
If the hadron mass is above threshold, it can strongly decay to $H_1 H_2$
as long as quantum numbers allow decay.

- Hadrons well bellow threshold: “easy”
Unfortunately none of exotic experimental candidates is found well below threshold.
- Hadron resonances above threshold and states slightly below threshold: *challenging*
 - until recently: so-called single-hadron approximation
 - now: rigorous treatment by determining scattering matrix for two hadrons $H_1 H_2$
- lattice searches for manifestly exotic hadrons (usually above several thresholds): *very challenging*

Lattice setup

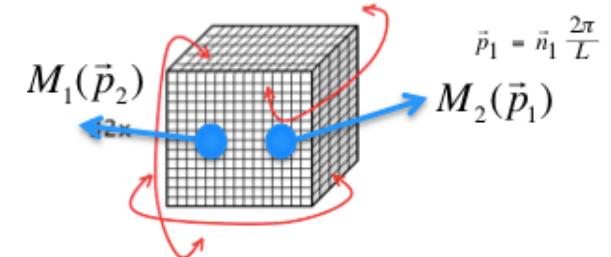
	PACS-CS	
	Ensemble (1)	Ensemble (2)
$N_L^3 \times N_T$	$16^3 \times 32$	$32^3 \times 64$
N_f	2	2+1
a [fm]	0.1239(13)	0.0907(13)
L [fm]	1.98(2)	2.90(4)
m_π [MeV]	266(3)(3)	156(7)(2)

- Wilson-clover quarks
- Fermilab method for c and b : $E_M(p) = M_1 + \frac{\mathbf{p}^2}{2M_2} - \frac{a^3 W_4}{6} \sum_i p_i^4 - \frac{(\mathbf{p}^2)^2}{8M_4^3} + \dots$;
[El Khadra, Kronfeld, Mackenzie, 1997]
Only splittings with respect to a chosen reference mass are compared to experiment.
- evaluating Wick contr.: distillation (Ensemble 1) [Pearson et. al., HSC, 2009]
stochastic distillation (Ensemble 2) [Morningstar et al., 2011]

Discrete energy spectrum from correlators

Example: meson channel with given J^{PC}

$$\mathcal{O} = \bar{q}\Gamma q, \quad (\bar{q}\Gamma_1 q)(\bar{q}\Gamma_2 q), \quad [\bar{q}\bar{q}][qq]$$



$$C_{ij}(t) = \langle 0 | \mathcal{Q}_i(t) \mathcal{Q}_j^+(0) | 0 \rangle = \sum_n Z_i^n Z_j^{n*} e^{-E_n t} \quad Z_i^n = \langle 0 | \mathcal{Q}_i | n \rangle$$

Energies and overlaps extracted using GEVP $C(t)u^{(n)}(t) = \lambda^{(n)}(t)C(t_0)u^{(n)}(t)$

All physical states with given J^{PC} appear as energy levels E_n in principle : single particle, two-particle,...

channel : "eigenstates"

$J^{PC} = 1^{--}, \bar{u}d$: $\rho, \pi\pi$

$J^{PC} = 1^{++}, \bar{c}c$: $\chi_{c1}, X(3872), DD^*$

$J^{PC} = 1^{+-}, \bar{c}c\bar{u}d$: $J/\psi, DD^*$

Two-meson states:

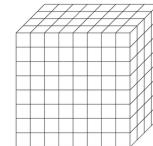
- In experiment: two-meson decay products with continuous E .
- On lattice: discrete E due to finite L and periodic BC: $p=n 2\pi/L$

Bound state and narrow resonance:

typically lead to extra energy level (in addition to two-meson levels)

States well below threshold: “easy” precision spectrum

- $m = E_n$ for $P=0$ $a \rightarrow 0$, $L \rightarrow \infty$, $m_q \rightarrow m_q^{\text{phy}}$
- Available from a number of lattice QCD collaborations for a number of years
- Only few examples shown



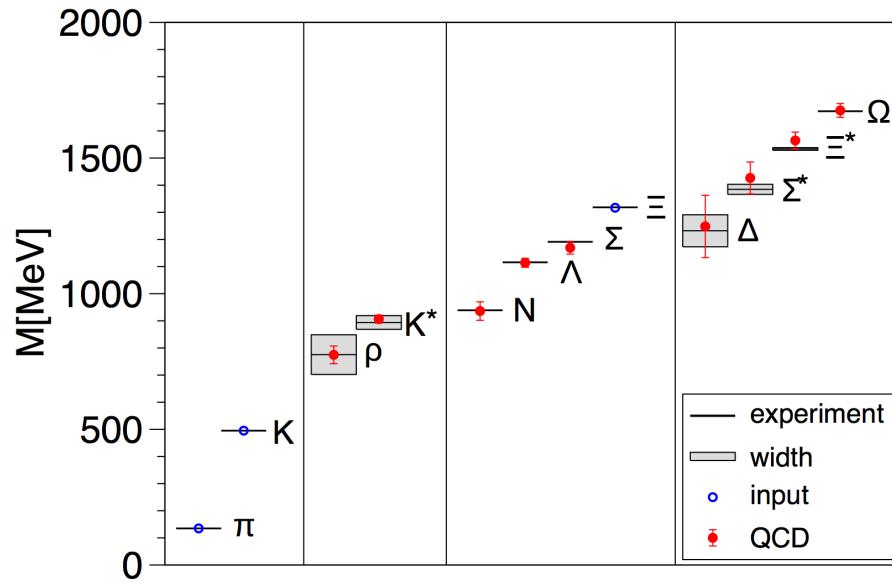
Proton and neutron constitute more than 99% of the bright side of universe

$$m_u c^2 \approx 2 \text{ MeV} \quad m_d c^2 \approx 5 \text{ MeV}$$

$$m_p c^2 \approx 938.3 \text{ MeV} \quad m_n c^2 \approx 939.5 \text{ MeV}$$



Higgs mechanism provides tiny contribution, the rest of visible mass in universe is due to the strong interaction in hadrons



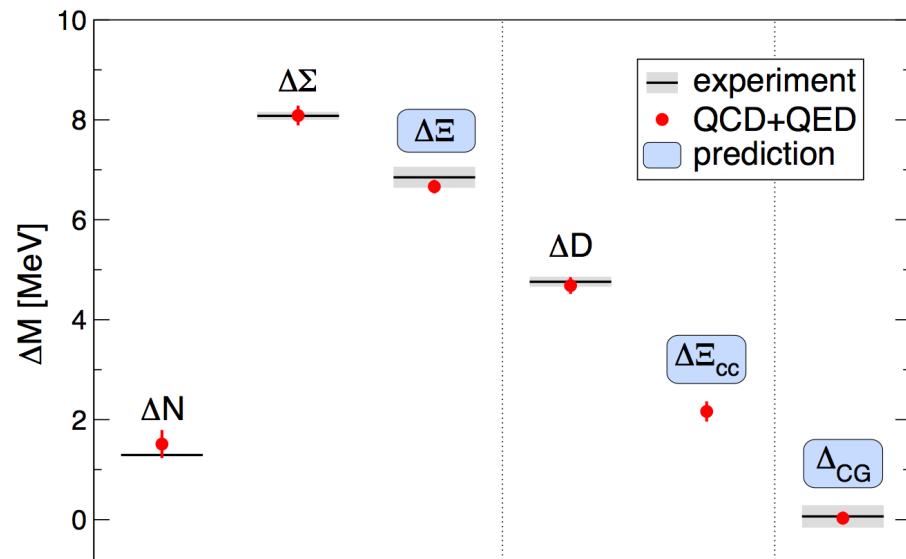
BMW collaboration
Science 322, 2008

QCD

$$m_u = m_d$$

$$m_n, m_p$$

S. Prelovsek, Fermilab 2015



$$m_n - m_p$$

BMW collaboration
Science 347, 2015

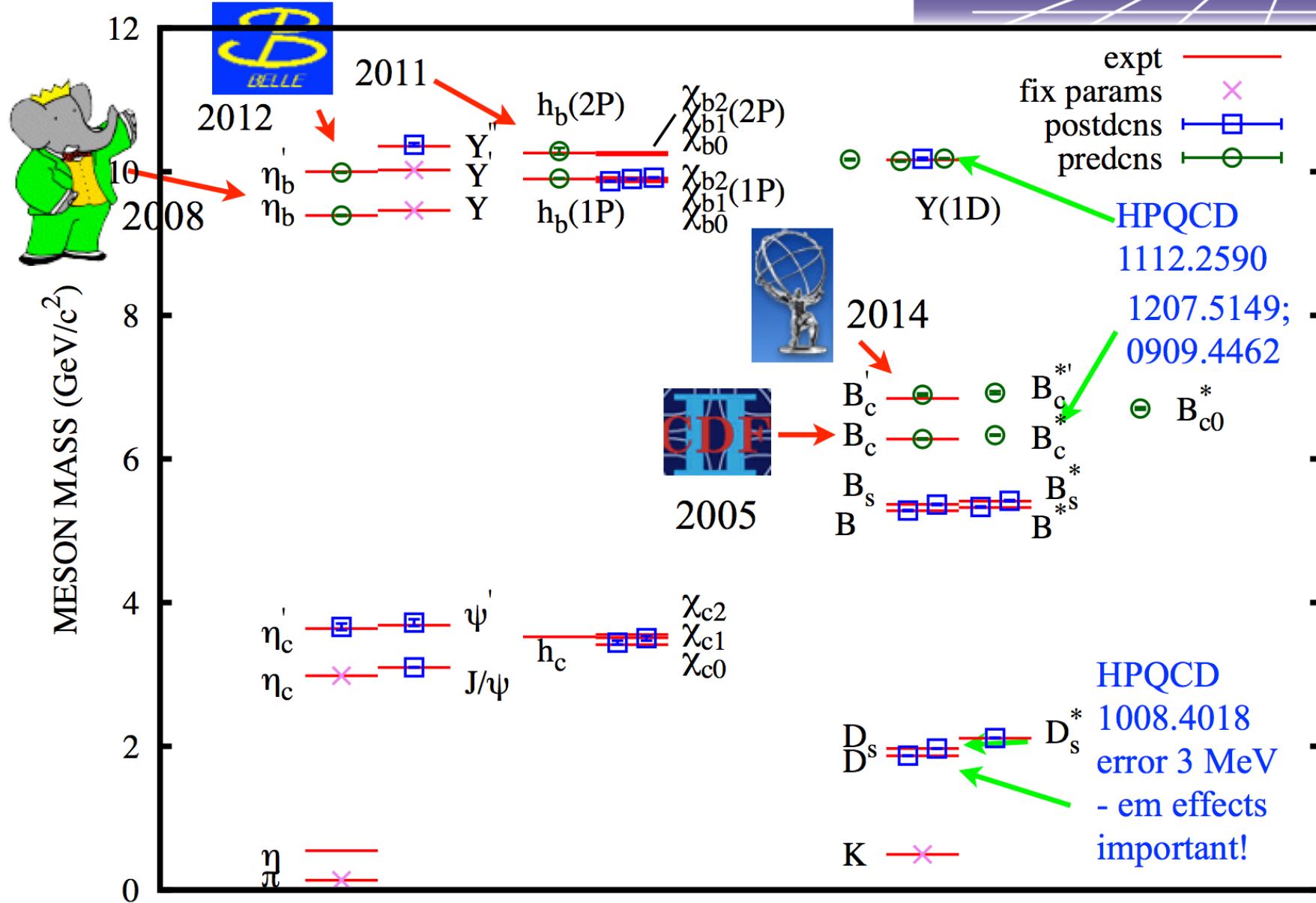
QCD + QED

$$m_u \neq m_d$$

$m=E_n$ The gold-plated meson spectrum

High Precision QCD

HPQCD



Hadrons near or above strong decay threshold

challenging

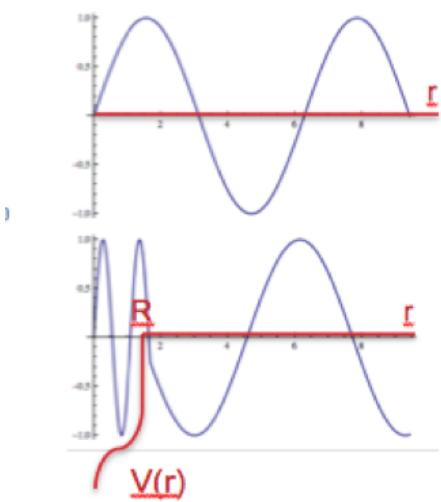
Single-hadron approximation used until few years ago:
ignores strong decays of resonances and effects of thresholds

I will concentrate on “**Rigorous approach**” which addresses
scattering of two hadrons in experiment or theory

Two-hadron scattering

Scattering phase shift δ

$$u(r) = r \psi(r)$$



QM interpretation:

$$\psi(r) \propto \frac{\sin(pr)}{r}$$

$$\psi(r) \propto \frac{\sin(pr + \delta)}{r} \quad r > R$$

unknown

$$\psi(r) \propto \frac{\sin(pr + \delta)}{r} \quad r < R$$

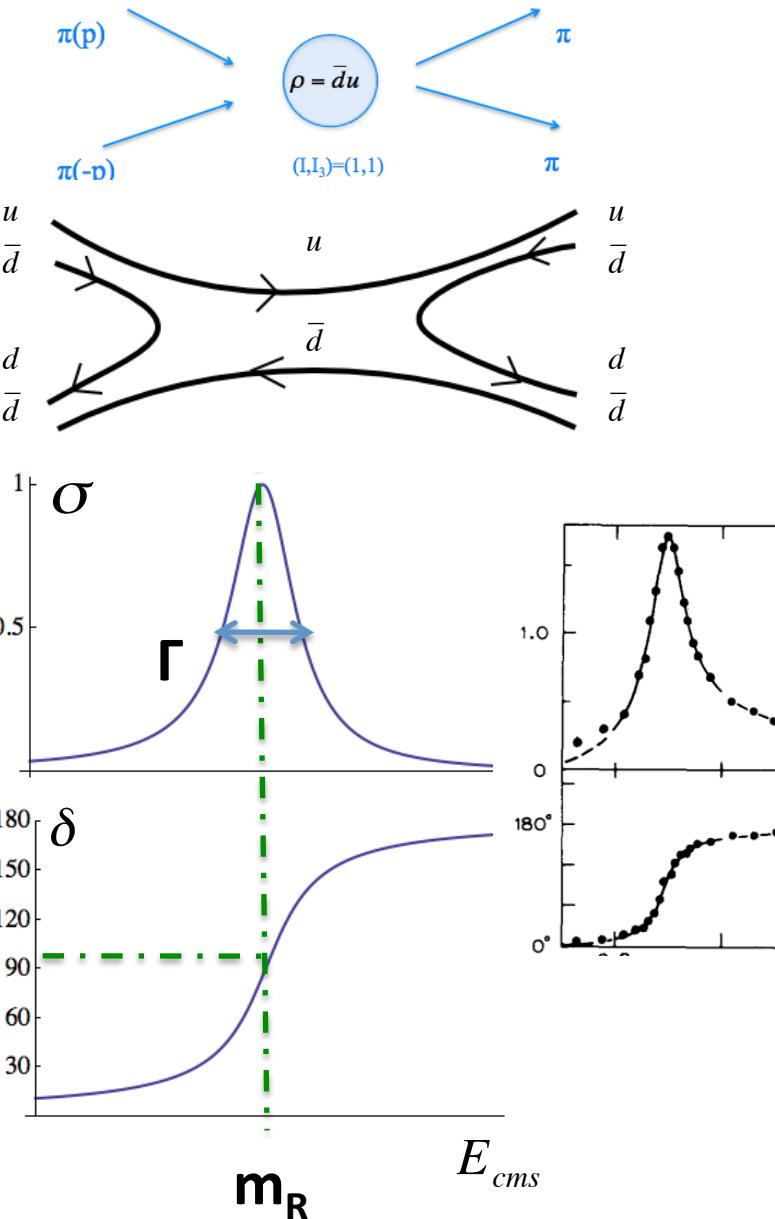
scattering matrix $T(E)$ from $\delta(E)$ for elastic scat:

$$S(E) = 1 + 2 i T(E) = e^{2 i \delta(E)}$$

$$T = \frac{e^{2i\delta} - 1}{2i} = \sin \delta \ e^{i\delta} = \frac{1}{\cot \delta - i}$$

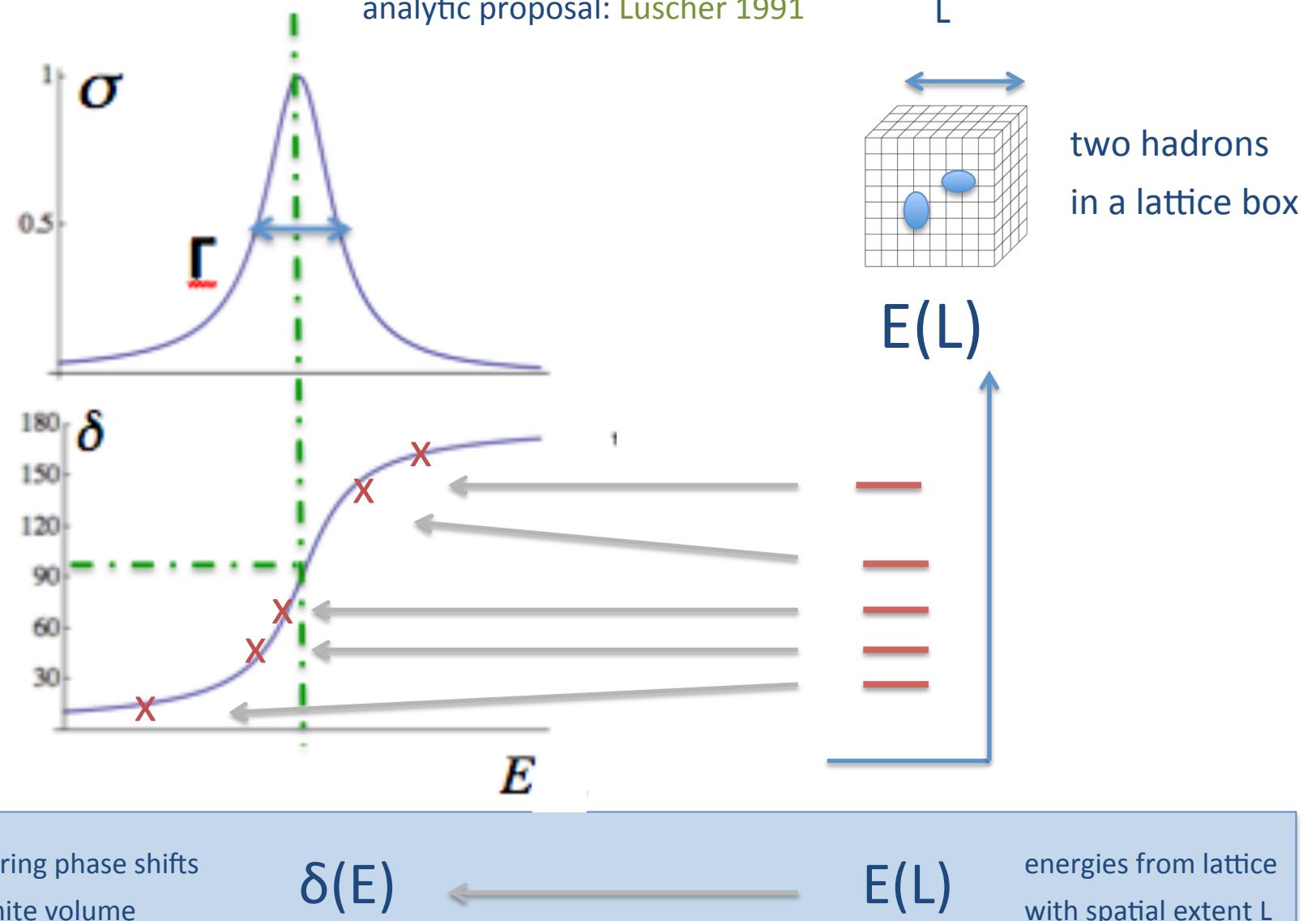
$$\sigma \propto |T|^2 = \sin^2 \delta$$

S. Prelovsek, Fermilab 2015



Rigorous treatment: scattering info from spectrum

analytic proposal: Luscher 1991



Relation between E and $\delta(E)$

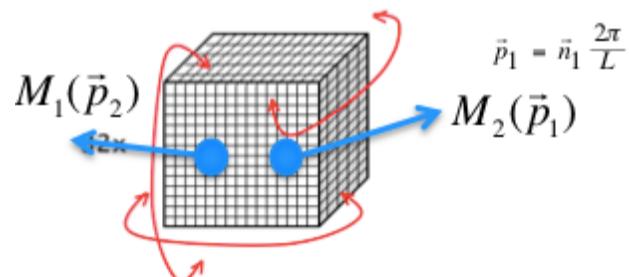
analytic proposal: Luscher 1991

Energies of two hadrons in a box:

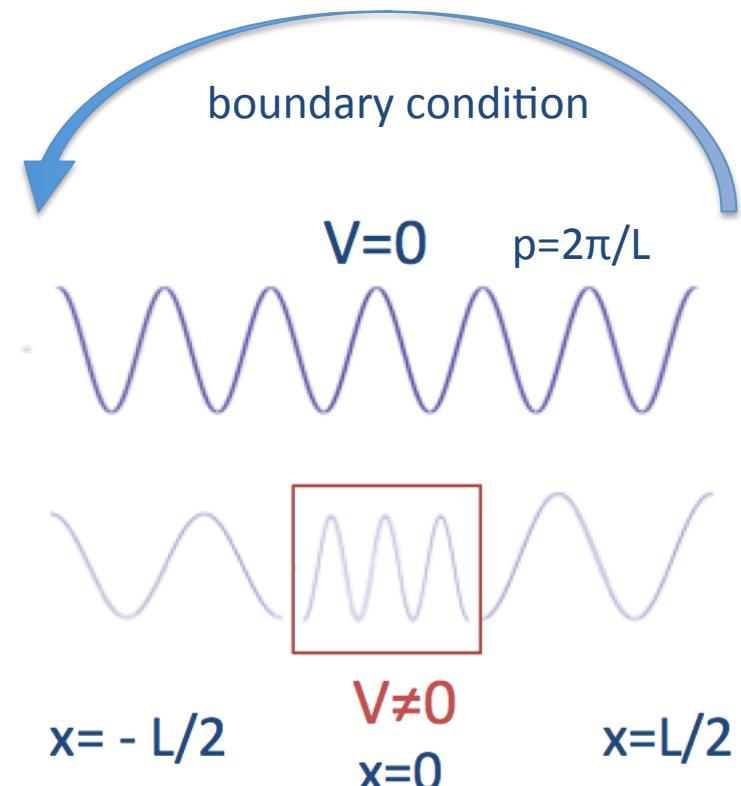
$$E(L) = \sqrt{m_1^2 + \vec{p}_1^2} + \sqrt{m_2^2 + \vec{p}_2^2} + \Delta E$$

- due to strong interaction
- gives rigorous info on δ

$$\vec{p}_{1,2} = \frac{2\pi}{L} \vec{n}$$



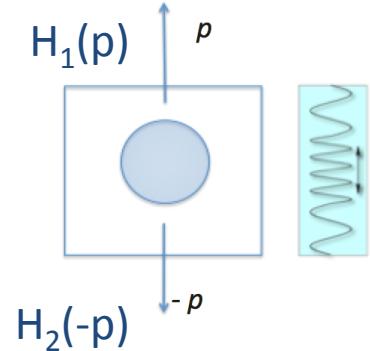
S. Prelovsek, Fermilab 2015



Scattering of two hadrons

elastic scattering with total momentum $P=0$: $E=E_{cm}$

$$E_n(L) \xrightarrow{\text{Luscher's eq.}} \delta(E)$$



Scattering matrix for partial wave l :

$$S(E) = e^{2i\delta(E)}, \quad S(E) = 1 + 2iT(E), \quad T(E) = \frac{1}{\cot \delta(E) - i}$$

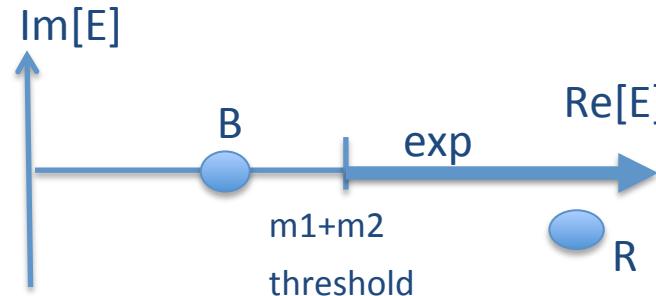
Bound state (B):

$$\cot[\delta(E_B)] = i, \quad E_B < m_1 + m_2$$

Resonance (R) (of Breit-Wigner type):

$$T(E) = \frac{-E \Gamma}{E^2 - m_R^2 + i E \Gamma}, \quad \Gamma(E) = g^2 \frac{p^{2l+1}}{E^2}$$

Locations of poles of $T(E)$
for res. and bound st.

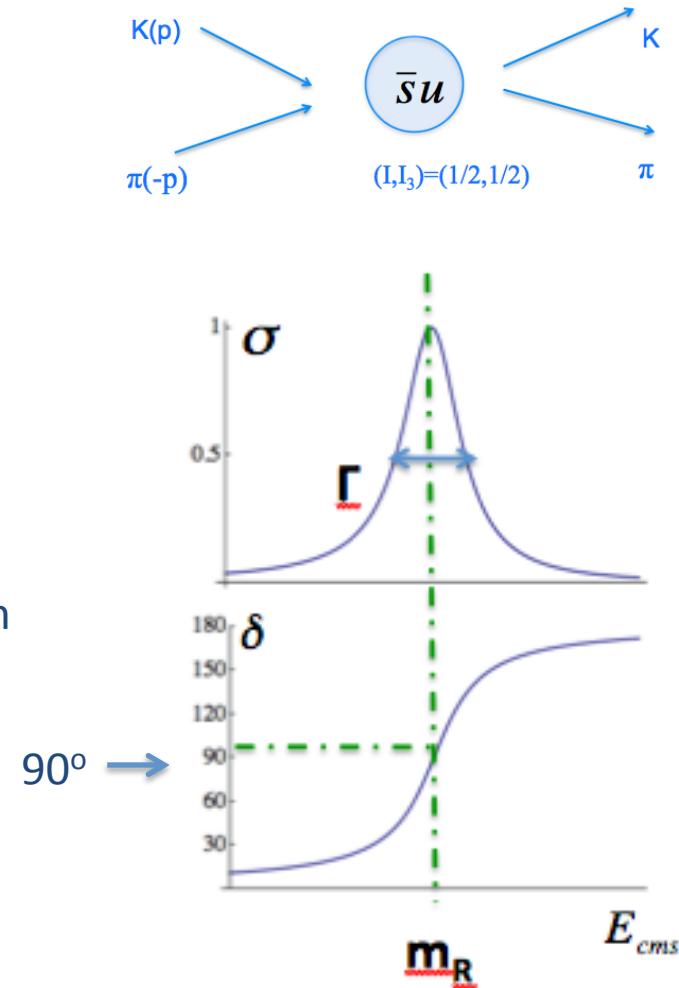


Resonances above threshold

Decay quickly through strong interaction

$$\tau = \hbar / \Gamma$$

$$\Gamma = 10 - 300 \text{ MeV}$$

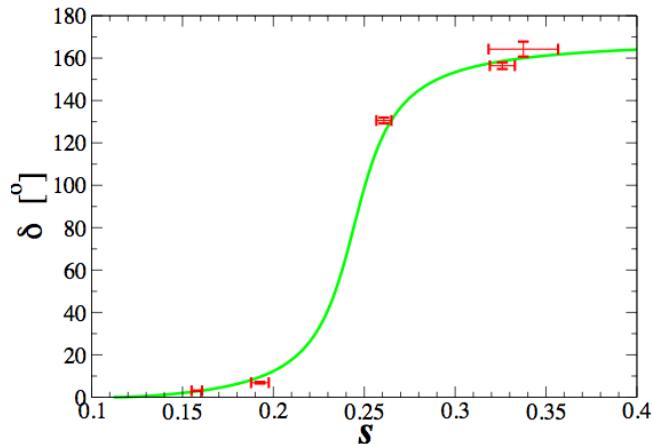


ρ resonance

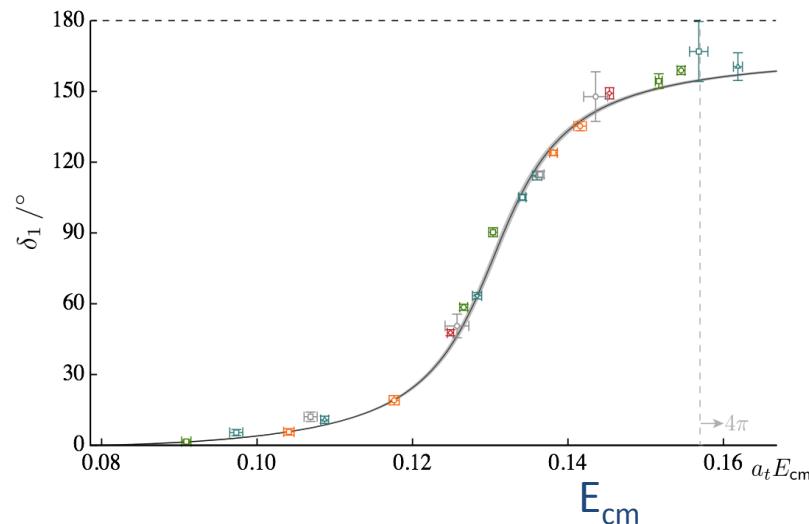
(the only “mature” hadronic resonance from lattice)

$\mathcal{O} : \bar{u}d, \pi\pi$

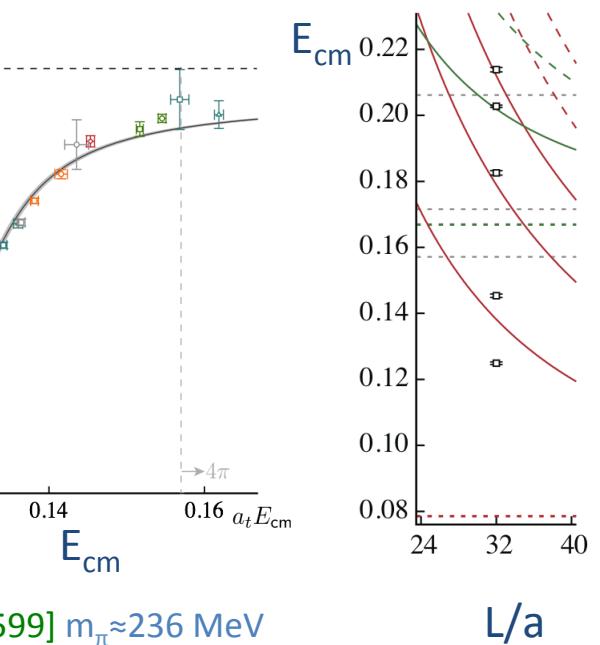
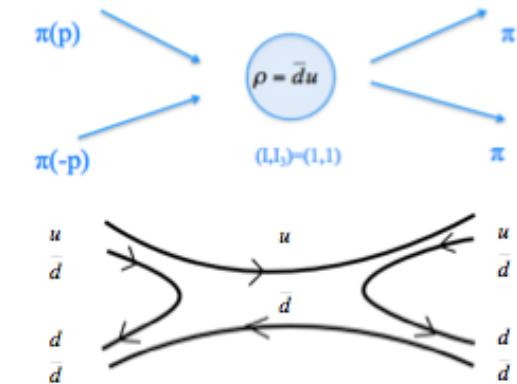
$$E_{cm} \rightarrow \delta(E_{cm})$$



Lang, Mohler, S.P., Vidmar, PRD 2011,
 $m_\pi \approx 266$ MeV



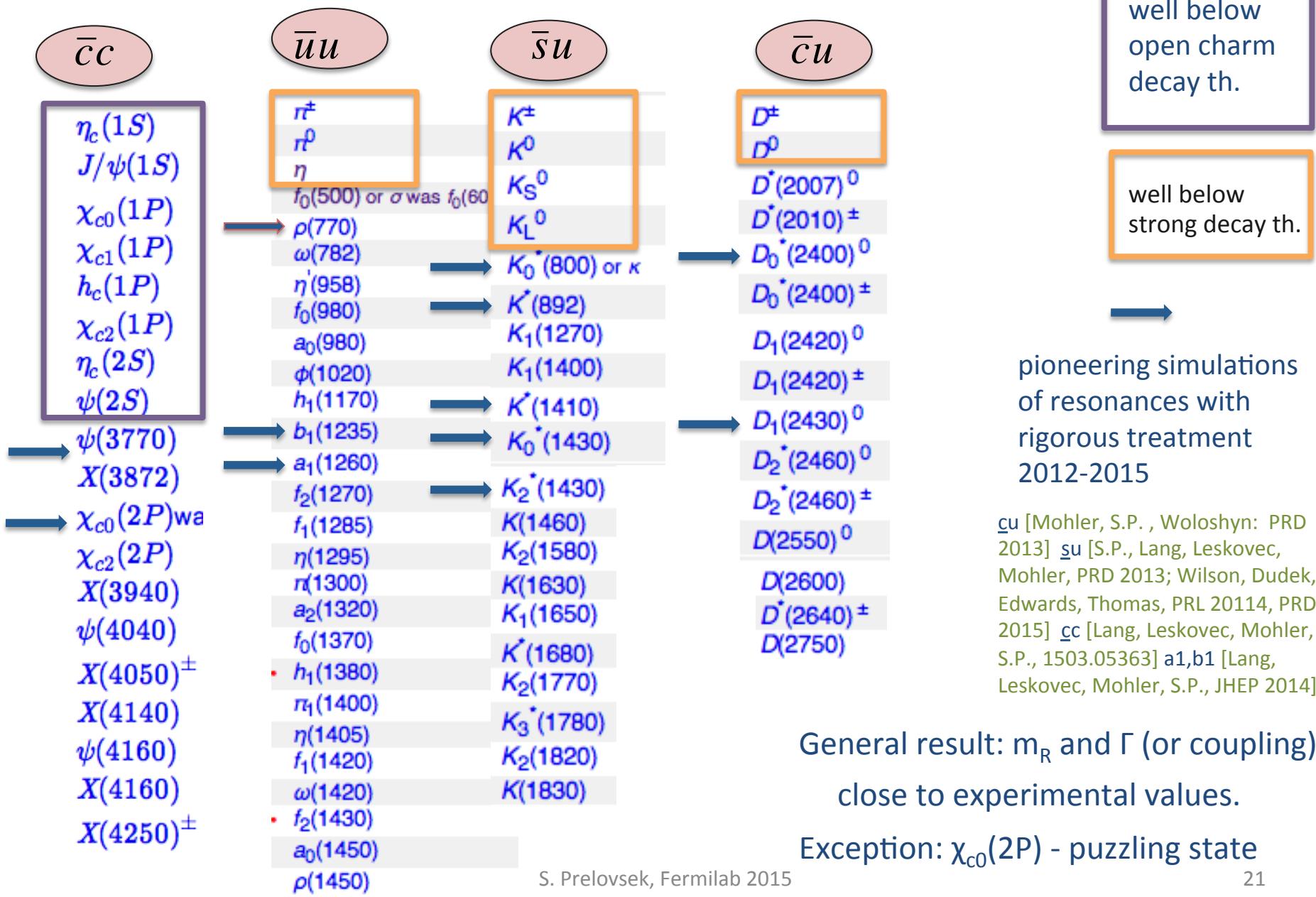
[HSC, PRD 1507.02599] $m_\pi \approx 236$ MeV



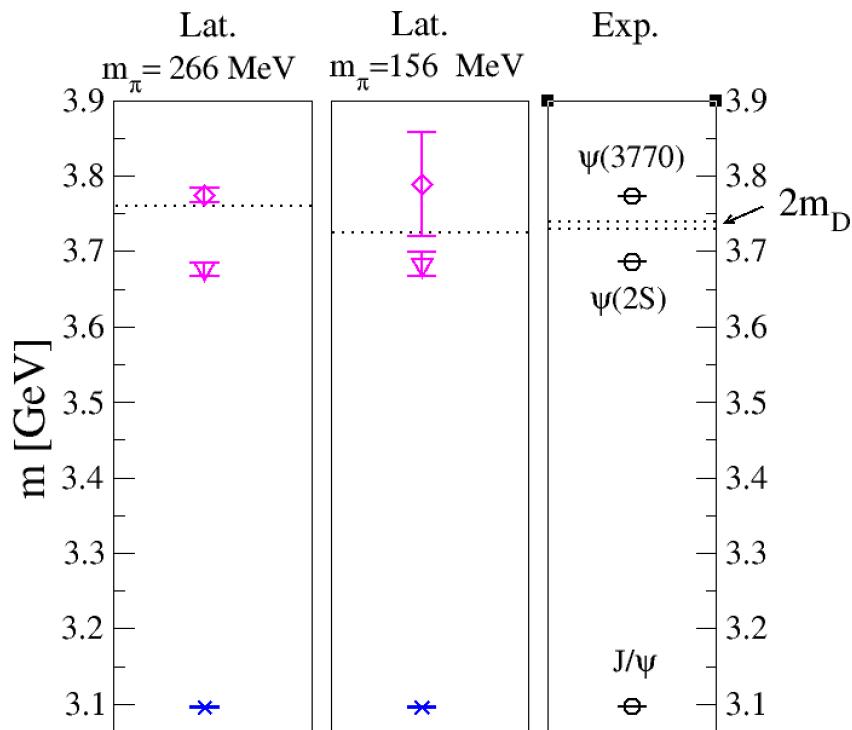
L/a

$$\text{BW: } \delta = \text{acot} \frac{m_R^2 - E_{cm}^2}{m_R \Gamma} \rightarrow m_R, \Gamma \text{ or } g_{\rho\pi\pi}$$

Other resonances simulated on the lattice



Resonance $\Psi(3770)$ and bound st. $\Psi(2S)$ from $D\bar{D}$ scattering in p-wave



$\mathcal{O}: \bar{c} c, D\bar{D}, J^{PC} = 1^{--}$

$D\bar{D}$ scat. in p-wave is simulated

T-matrix is determined from E_n

Fit of T-matrix gives:

BW resonance $\Psi(3770)$:

m_R (magenta diamonds)

Γ (given below)

Bound state $\Psi(2S)$ from pole in T:

m_B (magetna triangles)

$\eta_c(1S)$	$2m_D$
$J/\psi(1S)$	
$\chi_{c0}(1P)$	
$\chi_{c1}(1P)$	
$h_c(1P)$	
$\chi_{c2}(1P)$	
$\eta_c(2S)$	
$\psi(2S)$	
$\psi(3770)$	
$X(3872)$	
$\chi_{c0}(2P)_{wa}$	
$\chi_{c2}(2P)$	
$X(3940)$	
$\psi(4040)$	
$X(4050)^{\pm}$	
$X(4140)$	
$\psi(4160)$	
$X(4160)$	
$X(4250)^{\pm}$	

$\Psi(3770)$	Mass [MeV]	g (no unit)
Lat ($m_\pi=266$ MeV)	$3774 \pm 6 \pm 10$	19.7 ± 1.4
Lat ($m_\pi=156$ MeV)	$3789 \pm 68 \pm 10$	28 ± 21
Exp.	3773.15 ± 0.33	18.7 ± 1.4

$$\Gamma = \frac{g^2}{6\pi} \frac{p^3}{s}$$

Lang, Leskovec, Mohler, S.P.,
1503.05363

States slightly below threshold

$p\eta_c$ pentaquark

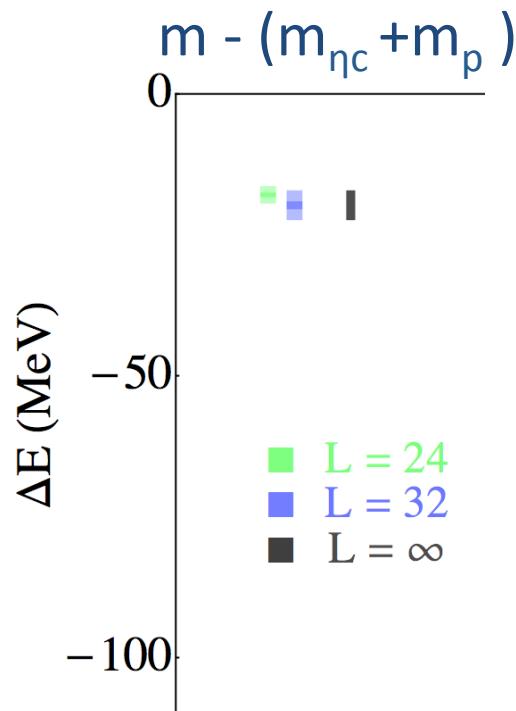
D_{s0}^* , D_{s1} , B_{s0}^* , B_{s1} , X(3872)

deuterium-like systems

Bound state of a η_c and p from lattice

[NPLQCD, 1410.7069, PRD, $m_\pi \sim 800$ MeV]

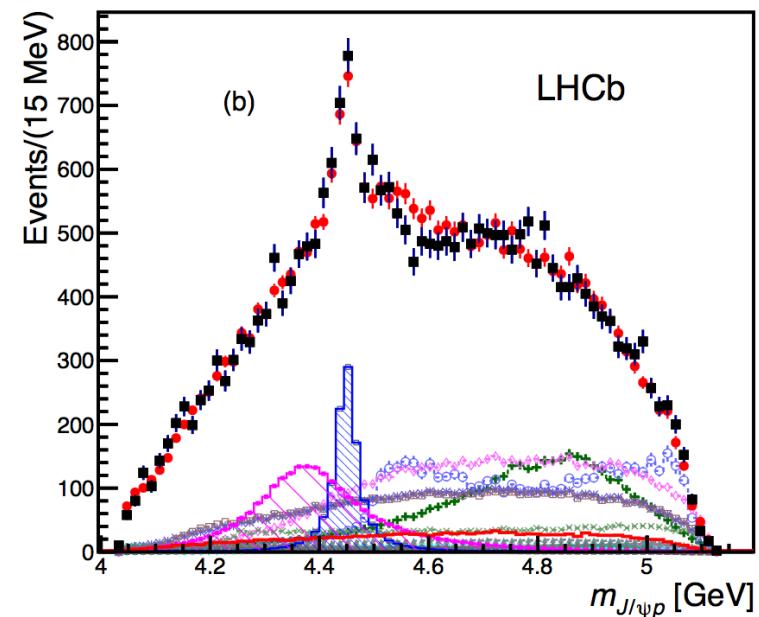
η_c p ~ 20 MeV below
 $\bar{c}c$ uud th. $m_{\eta_c} + m_p$



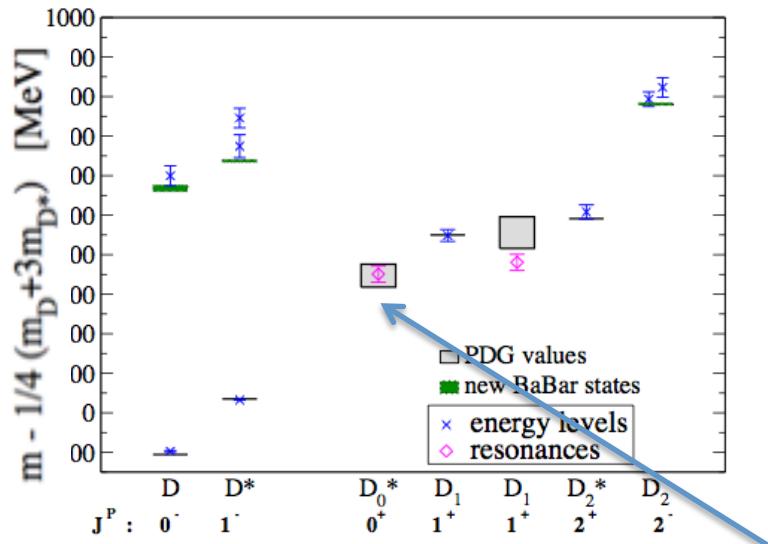
Two pentaquark resonances in J/ψ and p from exp

LHCb: 1507.03414

J/ψ p ~ 400 MeV above
 $\bar{c}c$ uud th. $m_{J/\psi} + m_p$



D_s and D scalar meson puzzle



D mesons (resonances)

scalar D resonance in $D\pi$

[D. Mohler, S.P., R. Woloshyn: PRD 2013]

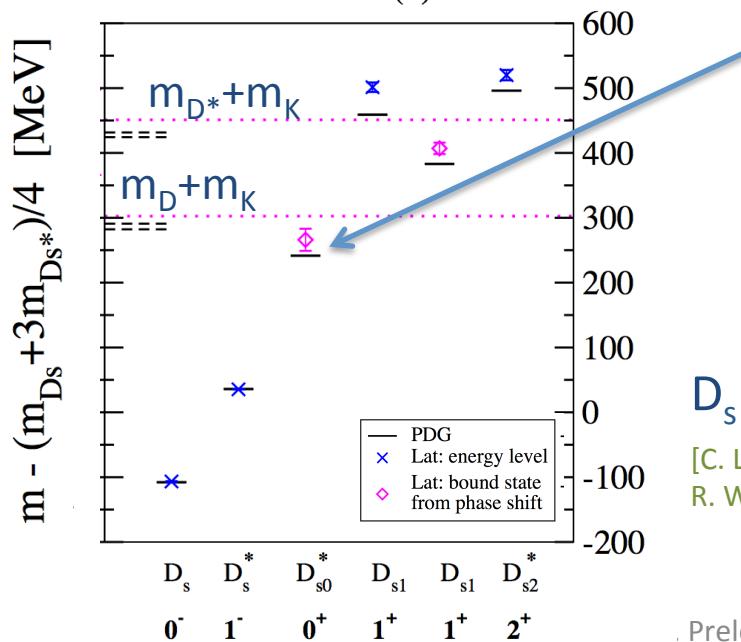
- why do these scalar partners have mass so close ?

$D_0^*(2400)$: $M \approx 2318$ MeV $\Gamma \approx 267$ MeV $\bar{c}u$ or $\bar{c}u\bar{s}s$?

$D_{s0}(2317)$: $M \approx 2318$ MeV $\Gamma \approx 0$ MeV $\bar{c}s$ or $\bar{c}s[\bar{u}u + \bar{d}d]$?

1) is D_0^* mass pushed up : valence ss pair ??

2) is D_{s0} mass pushed down : effect of DK threshold ??

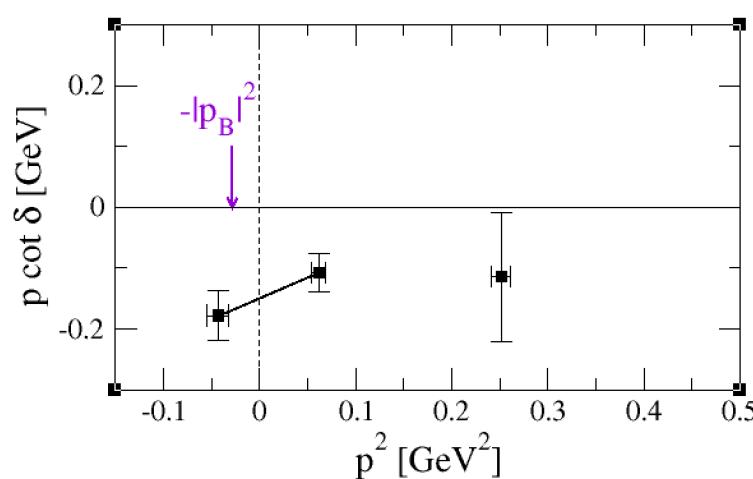
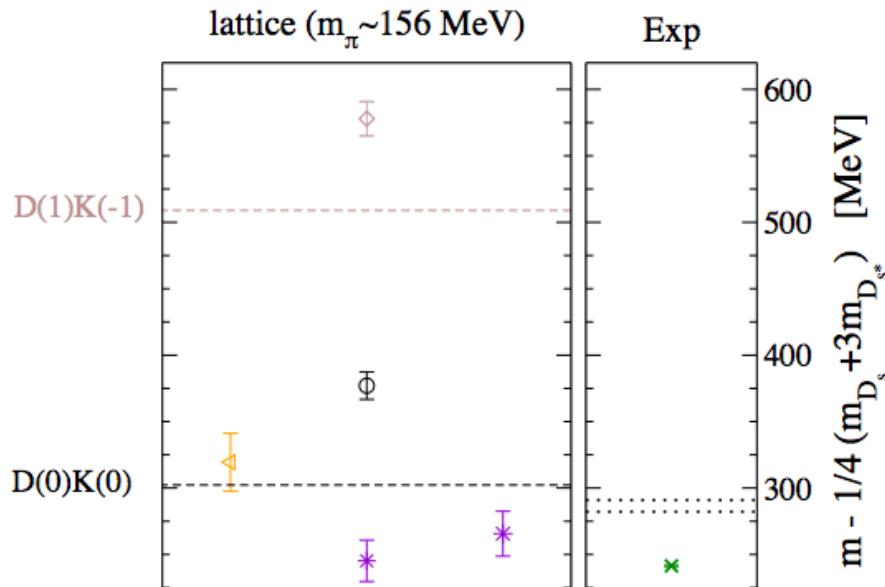


D_s mesons (near-threshold)

[C. Lang, L. Leskovec, D. Mohler, S.P., R. Woloshyn: PRL 2013, PRD 2014]

$D_{s0}^*(2317)$ and DK scattering

Energies from a lattice



- $E = E_K(p) + E_D(-p)$ renders p

- δ for DK scattering in s -wave extracted using Luscher's relation

$$p \cot \delta(p) = \frac{2\mathcal{Z}_{00}(1; (\frac{pL}{2\pi})^2)}{L\sqrt{\pi}}$$

- Interpolation: effective range

$$p \cot \delta(p) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2$$

$$a_0 = -1.33 \pm 0.20 \text{ fm}$$

$$r_0 = 0.27 \pm 0.17 \text{ fm}$$

$a_0 < 0$ indicates a state below th.

- pole position of $D_{s0}^*(2317)$

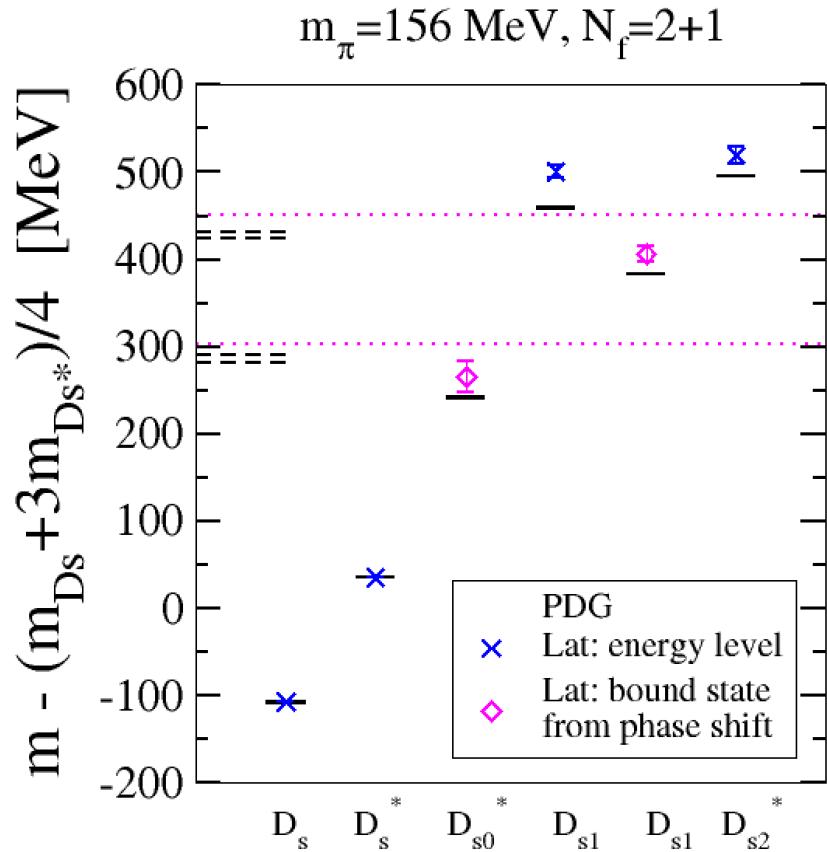
$$S \propto [\cot \delta - i]^{-1} = \infty, \quad \cot \delta(p_B) = i$$

$$K^{-1} = p_B \cot \delta = (i | p_B |) i = -| p_B |$$

$$m_{D_{s0}}^{lat, L \rightarrow \infty} = E_D(p_B) + E_K(p_B)$$

D. Mohler, C. Lang, L. Leskovec, S.P. , R. Woloshyn:
1308.3175, PRL : $m_\pi \approx 156$ MeV, $L \approx 2.9$ fm, Nf=2+1₂₆
sek, Fermilab 2015

D_{s0} and D_{s1} below DK and D^*K thresholds



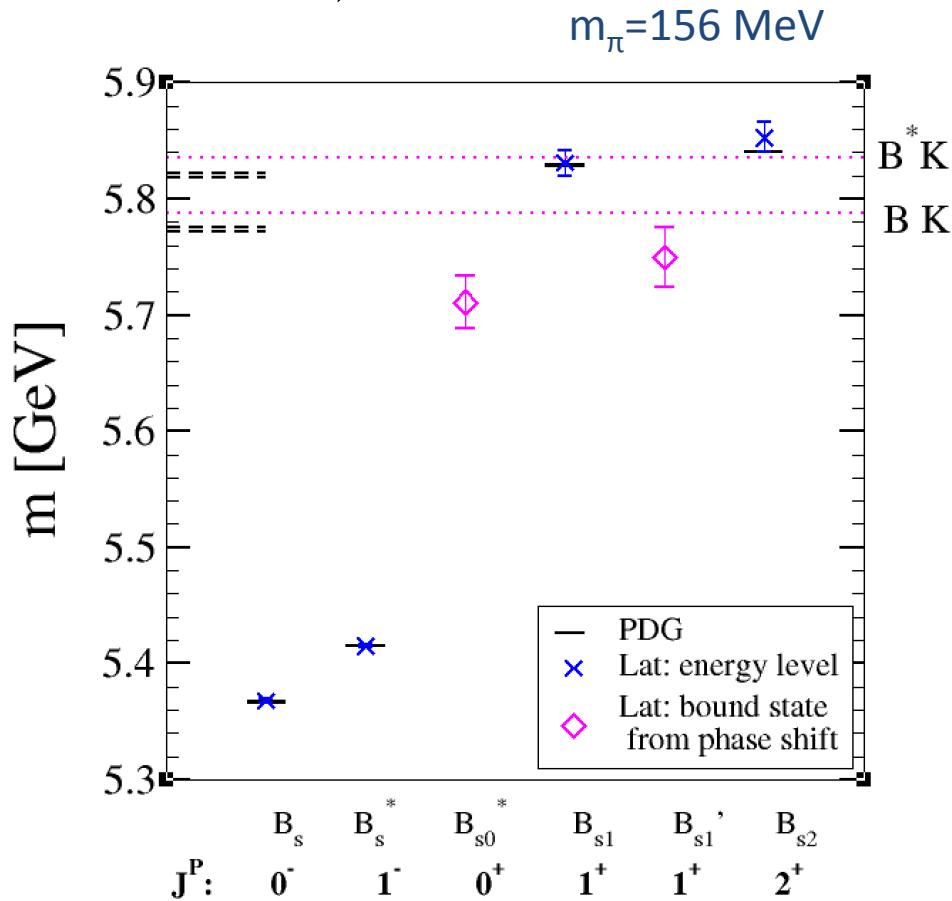
- D_{s0} and D_{s1} have been observed experimentally
- Quark models expected them above thresholds but they were found below them
- Our post-dictions agree with measured masses
- Threshold effect lowers their masses

- Composition of resulting D_{s0} and D_{s1} analyzed via Weinberg-type compositeness conditions:
[Martinez Torres, E. Oset, S.P., A. Ramos:
[1412.1706](#)]

[D. Mohler, C. Lang, L. Leskovec, S.P. , R. Woloshyn:
1308.3175, Phys. Rev. Lett 2013
1403.8103, PRD 2014]

Mass prediction for missing B_{s0} and B_{s1}

$$\mathcal{O} = \bar{s}b, BK$$



Quantities shown:

for two bound states :

$$m_B = (m_B - E_{th})^{lat} + E_{th}^{exp}$$

for other states :

$$m = (m - \bar{m})^{lat} + \bar{m}^{lat}$$

for dotted lattice thresholds :

$$E_{th} = (E_{th} - \bar{m})^{lat} + \bar{m}^{lat}$$

$$\bar{m} \equiv \frac{1}{4}(m_{Bs} + 3m_{Bs^*})$$

- B_{s1}' and B_{s2} agree well with exp

Predictions:

- B_{s0} bound state below BK th.
- B_{s1} bound state below B^*K th.

[C. Lang, D. Mohler, S.P.,

R. Woloshyn: 1501.0164]

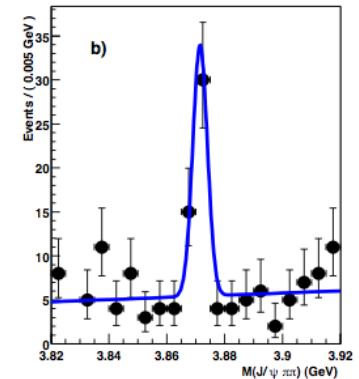
S. Prelovsek, Fermilab 2015

X(3872) , $J^{PC}=1^{++}$, charmonium-like

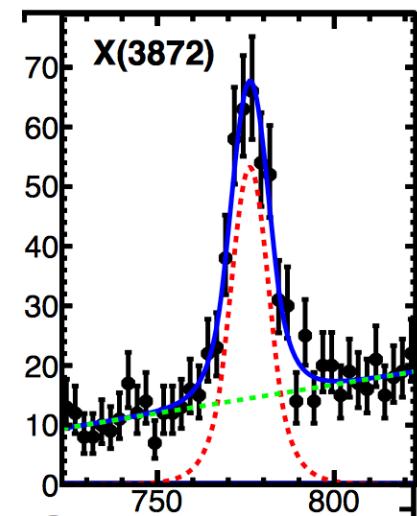
- First charmonium-like state discovered [Belle, PRL, 2003]
- sits within 1 MeV of $D^0\bar{D}^{0*}$ threshold
8 MeV below $D^+\bar{D}^{*-}$ threshold } isospin breaking effects may be important
- believed to have a large molecular $D^0\bar{D}^{0*}$ Fock component
- $\Gamma < 1.2$ MeV
- decays to $I=0, 1$ equally important

$$X(3872) \rightarrow J/\Psi \omega \text{ (} I=0 \text{)}$$

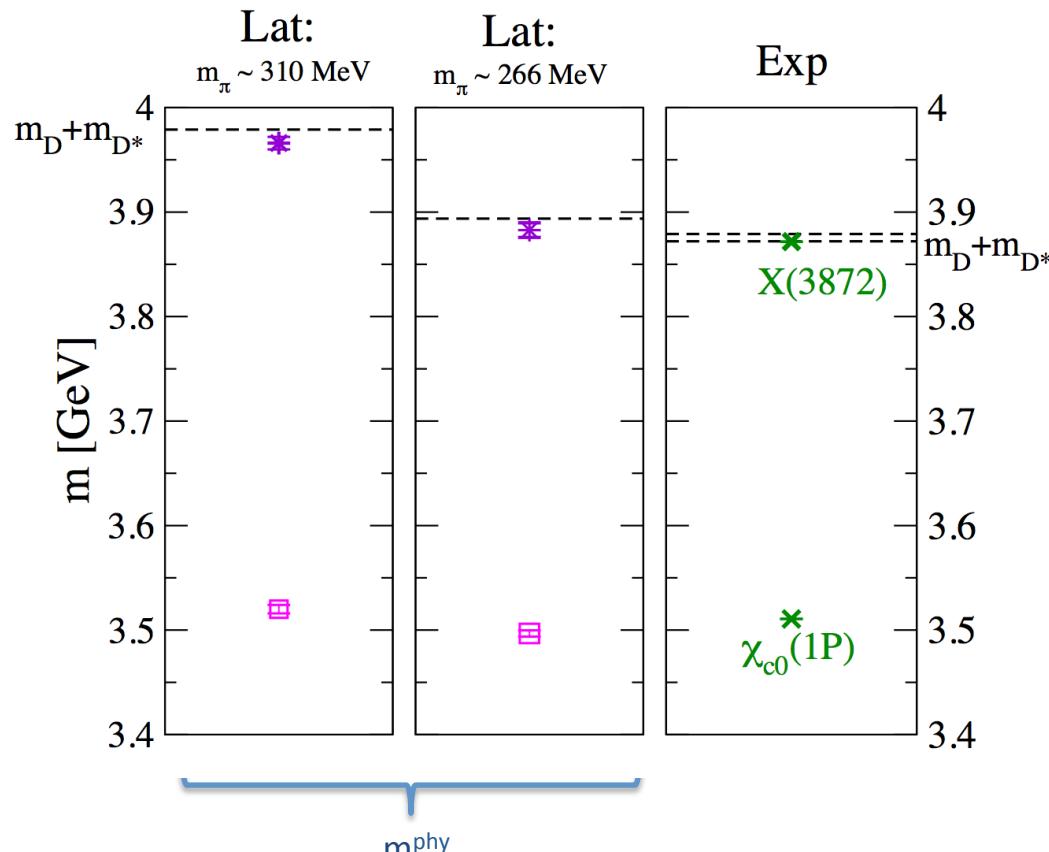
$$X(3872) \rightarrow J/\Psi \rho \text{ (} I=1 \text{)}$$



[LHCb, PRL 2013]



X(3872) as bound state from $\underline{D}\underline{D}^*$ scattering, $J^{PC}=1^{++}$, $I=0$



$\mathcal{O}: \bar{c} c, D\bar{D}^*$

- ground state: $\chi_{c1}(1P)$
- $\underline{D}\underline{D}^*$ scattering matrix near th. determined
- A pole of $T \propto \frac{1}{\cot \delta - i} = \infty$ found just below th. (violet star)
- The pole attributed to X(3872), which is a shallow bound state in both simulations
- Position of DD* threshold depends on $m_{u/d}$, and may be affected by discretization effects related to charm quark

Lattice evidence for X(3872):

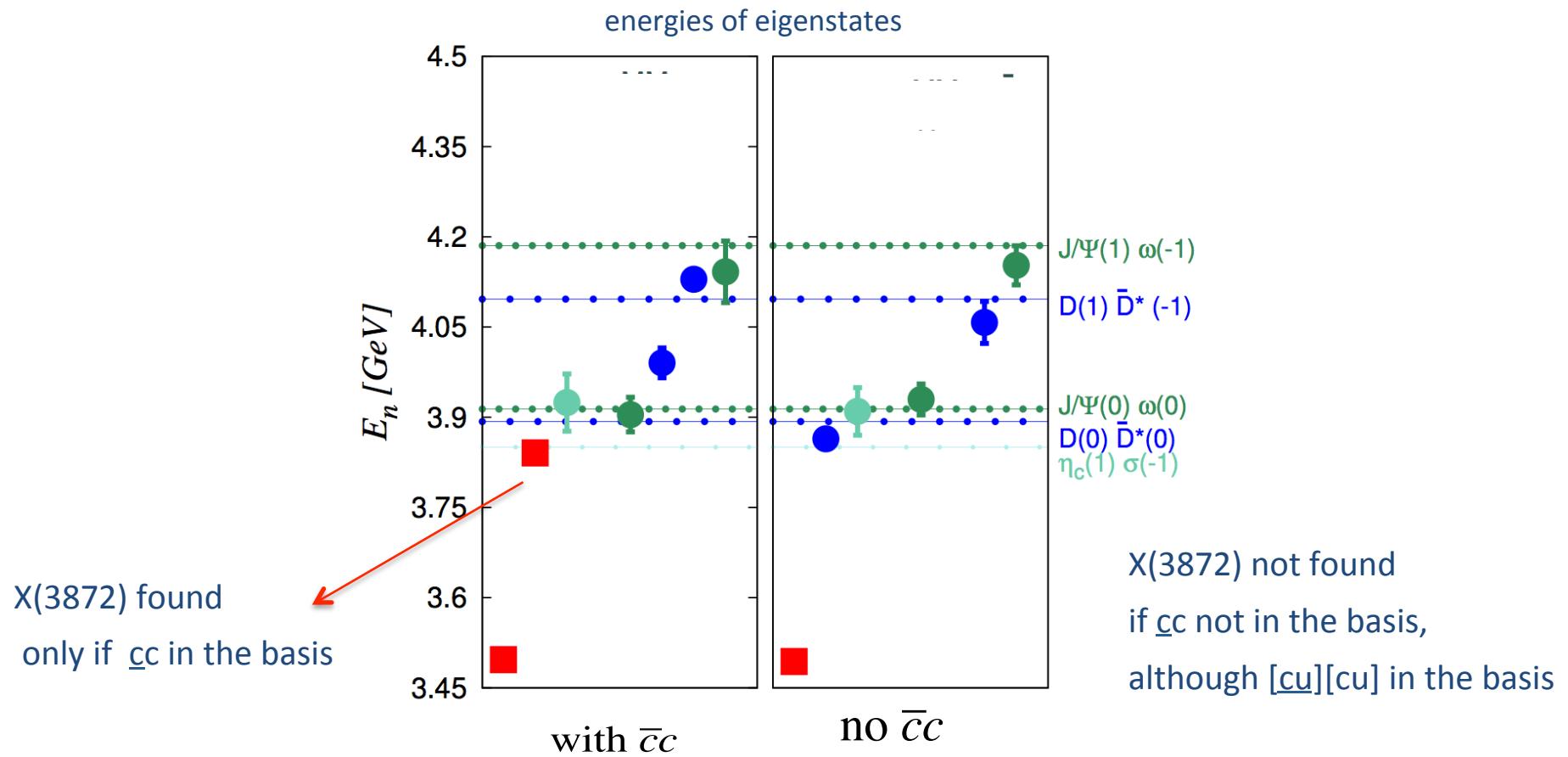
X(3872)	$m - (m_{D0} + m_{D0^*})$
lat ($m_\pi=310$ MeV)	- 13 \pm 6 MeV
lat ($m_\pi=266$ MeV)	- 11 \pm 7 MeV
exp	- 0.14 \pm 0.22 MeV

- $m_\pi \approx 266$ MeV, $a=0.124$ fm, $L=2$ fm
[S.P. and Leskovec: 1307.5172, PRL 2013]
- $m_\pi \approx 310$ MeV, $a=0.15$ fm, $L=2.4$ fm , HISQ
[Lee, DeTar, Na, Mohler , update of proc 1411.1389]

Which Fock components are essential for X(3872) with $I=0$?

$J^{PC}=1^{++}$ $\mathcal{O}:$ $\overline{c} c, \quad D\bar{D}^*, \ J/\psi\omega, \ \chi_{c1}\eta, \ \eta_c\sigma, \quad [\overline{cu}]_{3c}[cu]_{3c}, \ [\overline{cu}]_{6c}[cu]_{6c}$

essential do not seem not essential



[M. Padmanath, C.B. Lang, S.P.,

1503.03257]

S. Prelovsek, Fermilab 2015

Searches for hadrons with exotic flavor very challenging

$$Z_c = \bar{c}c\bar{d}u, \quad Z_b = \bar{b}b\bar{d}u, \quad P_c = \bar{c}cuud$$

few searches

to challenging

to recent

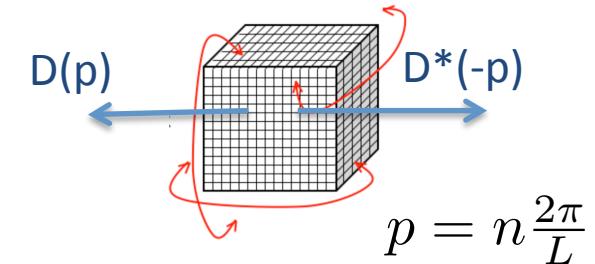
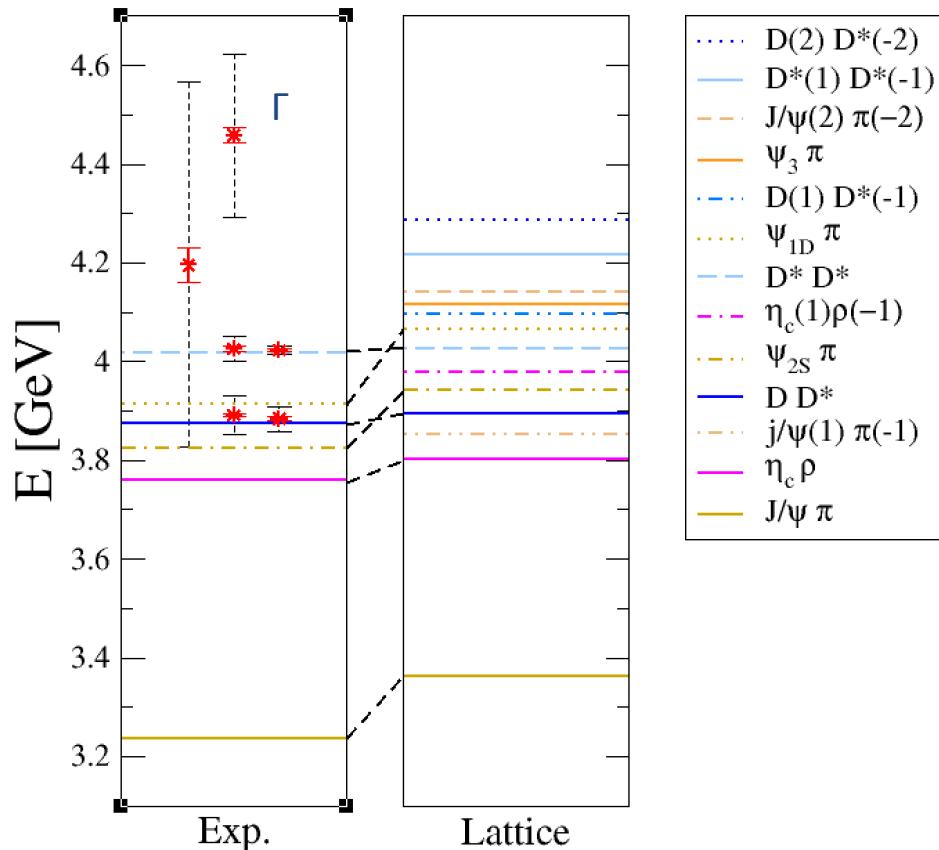
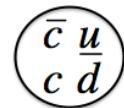
Even more challenging since most of experimental exotic states

- are above several thresholds and decay to several two-meson final states
- require simulation of coupled-channels

Coupled-channel scattering matrix extracted on lattice so far only in

- a-la Luscher: $K\pi$, $K\eta$ system [Willson, Dudek, Edward, Thomas, HSC, PRL 2014]

Z_c^+ channel : $|G=1^+, J^{PC}=1^{+-}$



Lattice:

Horizontal lines represent
energies of 13 two-meson states
in non-interacting case

$$E = E[M_1(p_1)] + E[M_2(p_2)]$$

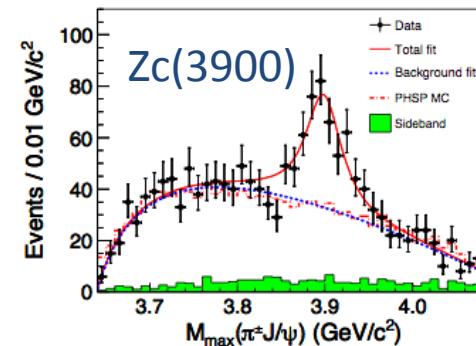
Extracting 13 two-meson states
is a challenge

$$\mathcal{O} : (\bar{c}u)(\bar{d}c), (\bar{c}c)(\bar{d}u), [\bar{c}\bar{d}][cu]$$

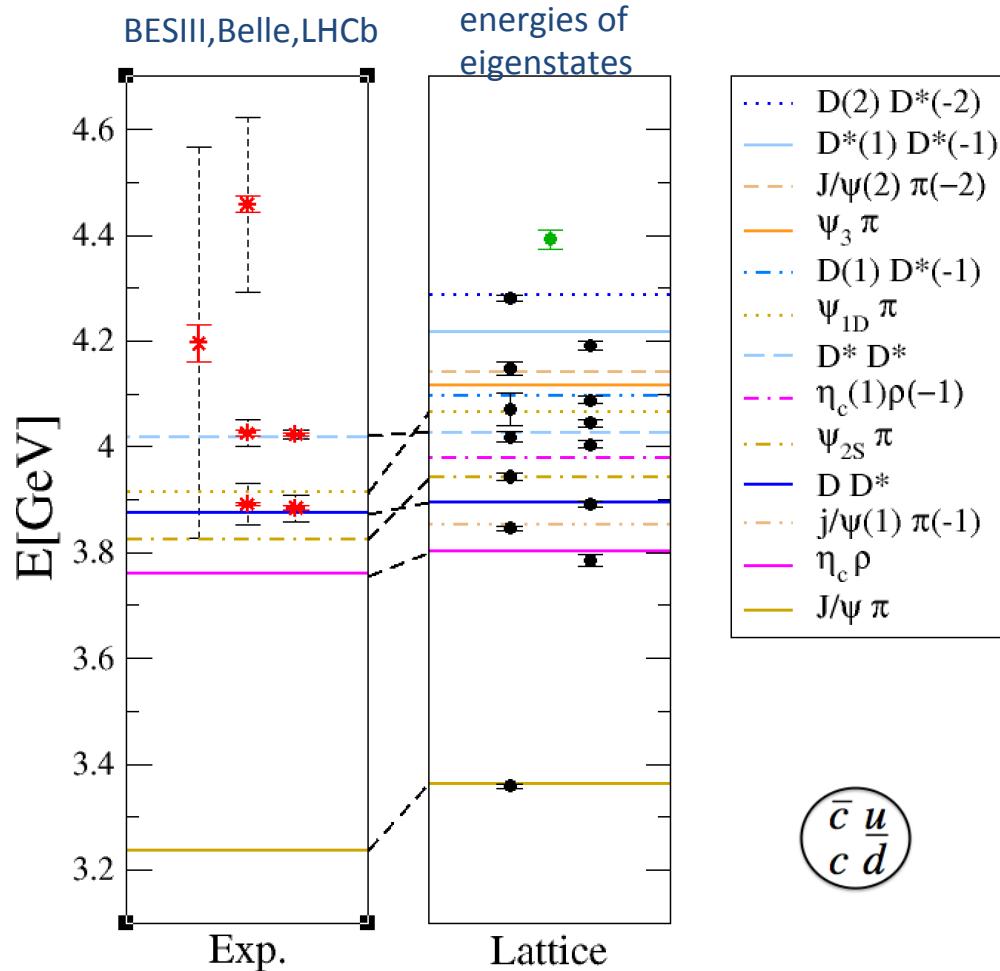
[S.P., Lang, Leskovec, Mohler, 1405.7612, PRD 2015]

Ensemble (2), $m_\pi \approx 266$ MeV, $L \approx 2$ fm, $N_f = 2$

S. Prelovsek, Fermilab 2015



Z_c^+ channel : $|G=1^+, J^{PC}=1^{+-}$



[S.P., Lang, Leskovec, Mohler, 1405.7612, PRD 2015]

similar conclusion [S.-H. Lee, C. DeTar, H. Na, 1411.1389]

$$\mathcal{O} : (\bar{c}u)(\bar{d}c), (\bar{c}c)(\bar{d}u), [\bar{c}\bar{d}][cu]$$

Lattice results:

- 13 expected two-meson eigenstates found as expected (black circles)
- no additional eigenstate below 4.2 GeV
- no candidate for Z_c^+ below 4.2 GeV

Exp:

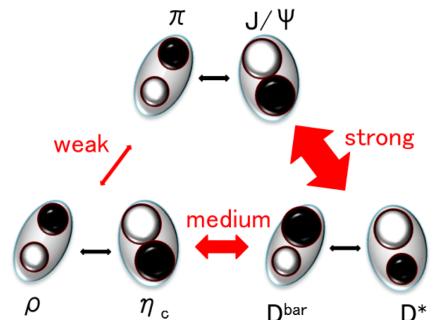
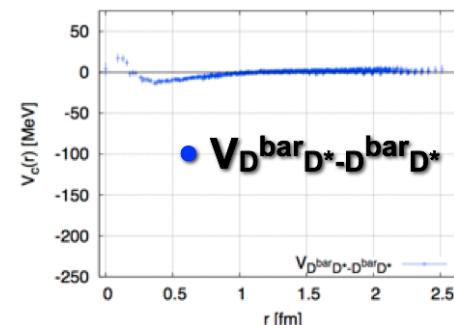
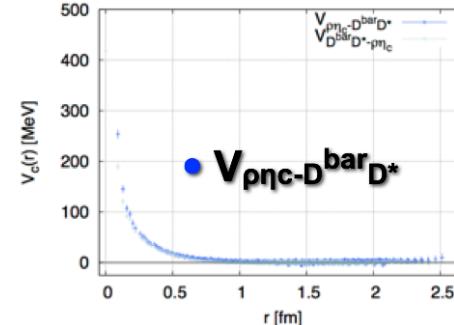
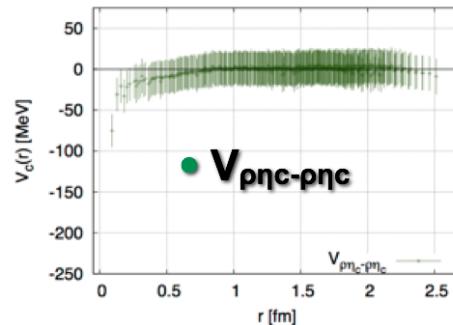
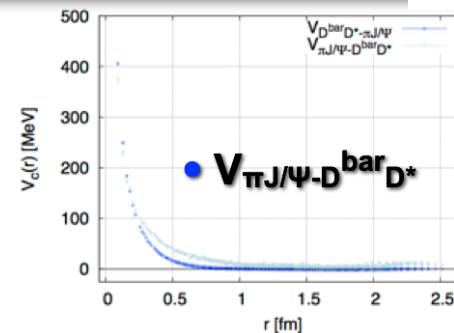
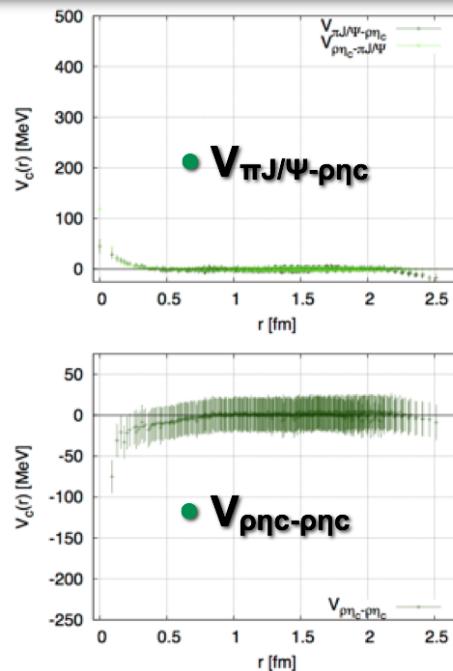
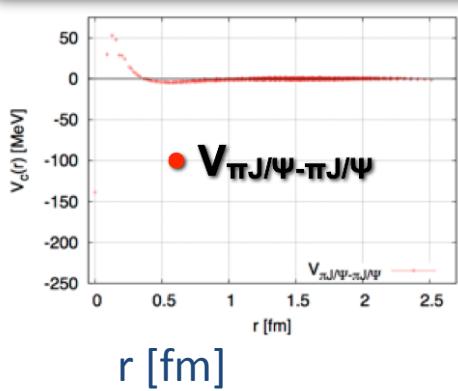
- $Z_c^+(3900)$ confirmed by three exp

Puzzle:

- why no eigenstate for $Z_c(3900)$?
- it would be naively expected if $Z_c(3900)$ related to a resonance pole
- is $Z_c(3900)$ of a different origin?
- perhaps coupled channel effect?

Z_c^+ channel : $|G=1^+, J^{PC}=1^{+-}|$

- HALQCD method [application on H-dibaryon: HALQCD, 1504.01717]



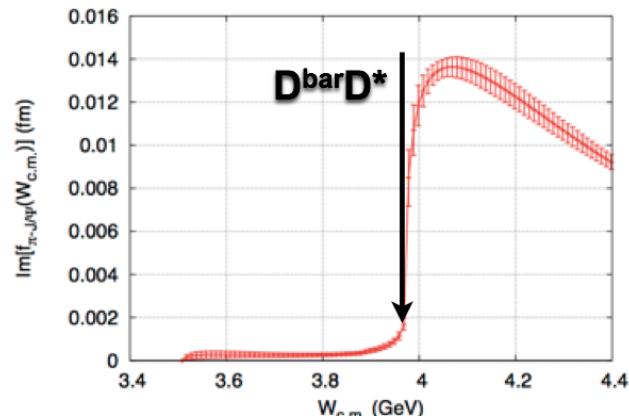
indication for
coupled channel eff.

HALQCD coll, Y. Ikeda et al $m_\pi \approx 410$ MeV, Nf=2+1+1
[private communication with Y. Ikeda]

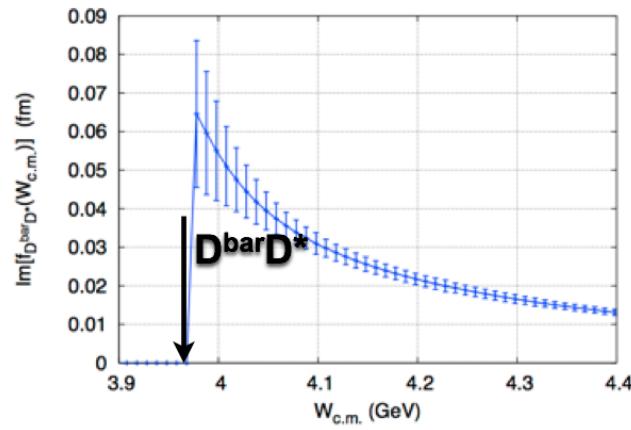
Z_c^+ channel: $|G=1^+, J^{PC}=1^{+-}$

Lattice quantity related to cross-section

- **$\pi J/\Psi$ invariant mass ($m_\pi = 410\text{ MeV}$)**

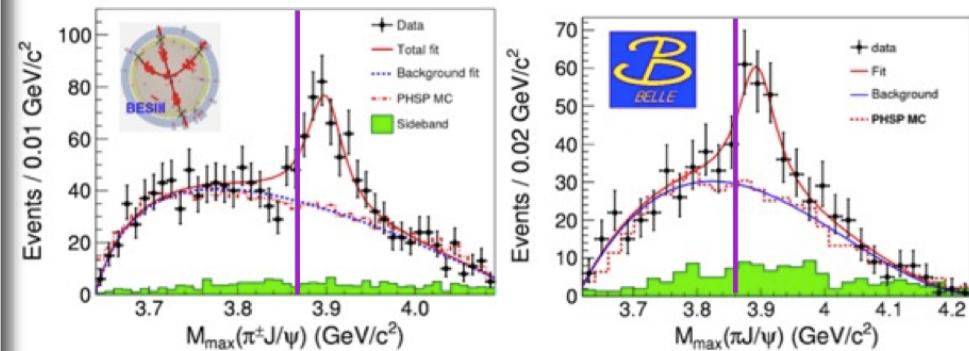


- **$D\bar{D}^*$ invariant mass ($m_\pi = 410\text{ MeV}$)**



Experimental cross-section related to $Z_c^+(3900)$

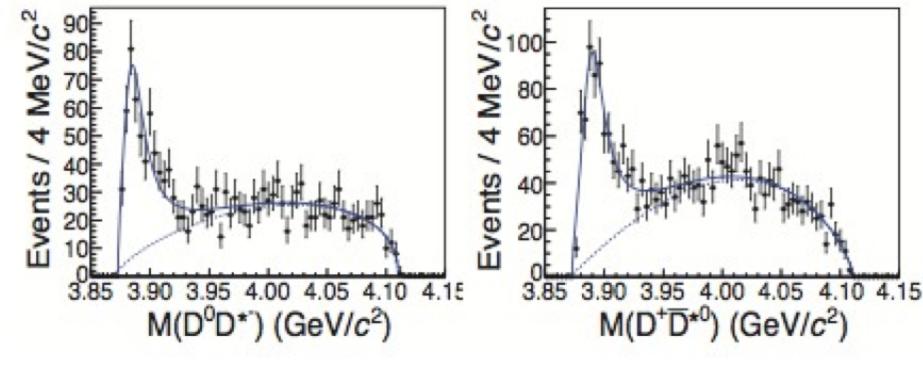
- **$e^+e^- \rightarrow \pi(\pi J/\Psi) @ 4.26\text{ GeV}$**



[BESIII Coll., PRL110 \(2013\).](#)

[Belle Coll., PRL110 \(2013\).](#)

- **$e^+e^- \rightarrow \pi^{+-} (D\bar{D}^*)^{-+}$**



[BESIII Coll., PRL112 \(2014\).](#)

HALQCD coll, Ikeda et al $m_\pi \approx 410$ MeV, Nf=2+1+1

[private communication with Y. Ikeda]

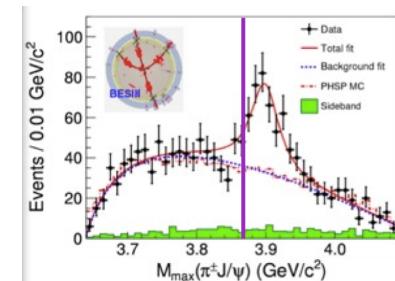
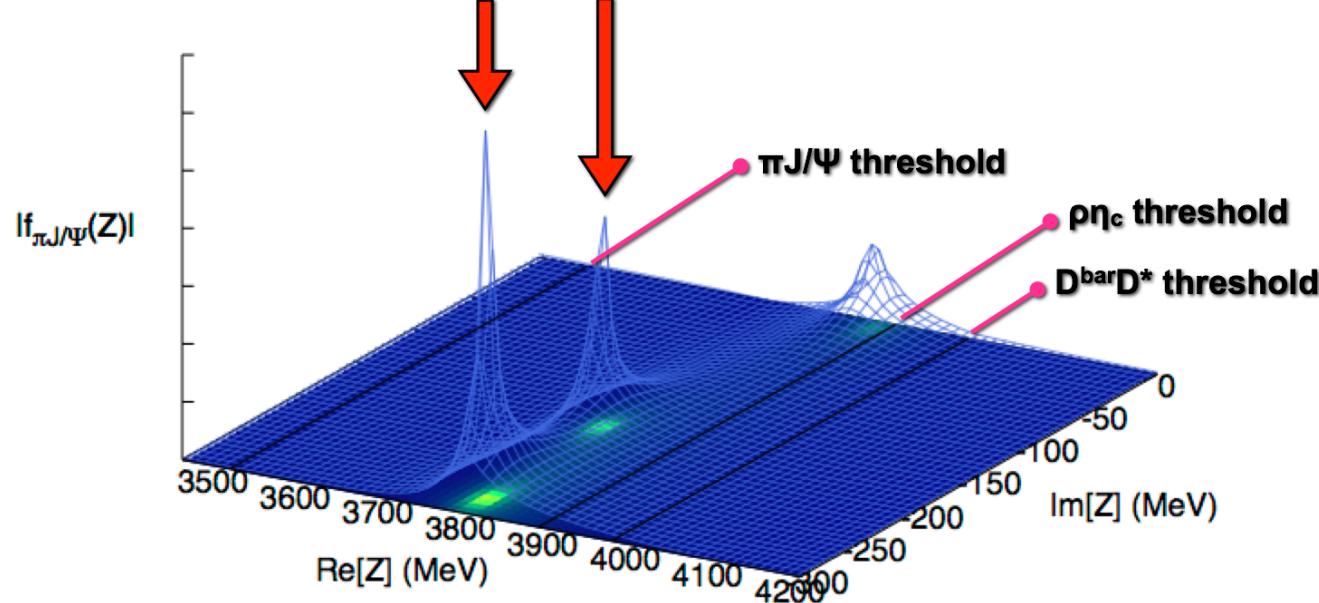
S. Prelovsek, Fermilab 2015

Lineshapes resemble
experimental $Z_c(3900)$.

DD*

Z_c^+ channel : $|G=1^+, J^{PC}=1^{+-}|$

Poles of S-matrix



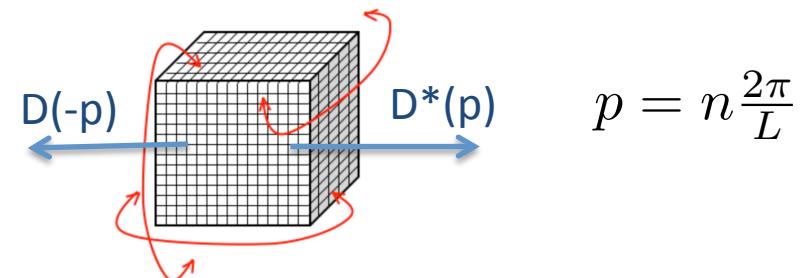
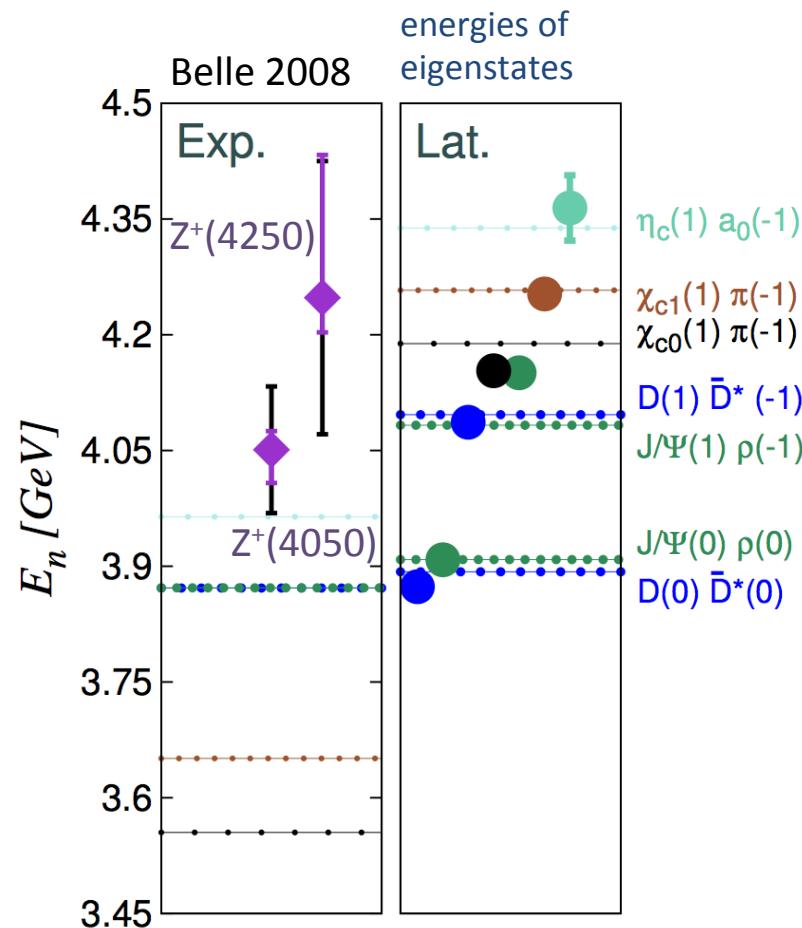
- exp: $Z_c^+(3900)$ peak appears above DD^* th.
- HALQCD: poles found BELOW DD^* th.
pole NOT interpreted as a resonance (such a pole would be expected above DD^*)
- Remains to be seen if HALQCD result is consistent with absence of Z_c eigenstate in S.P. et al, PRD 2015
- Scattering matrix has not yet been extracted from Luscher approach

HALQCD coll, Ikeda et al $m_\pi \approx 410$ MeV, Nf=2+1+1

[private communication with Y. Ikeda]

S. Prelovsek, Fermilab 2015

charged partner of X(3872); channel $I^G=1^-$, $J^{PC}=1^{++}$, $\underline{c}\underline{c}\underline{d}\underline{u}$



$$\mathcal{O} : (\bar{c}u)(\bar{d}c), (\bar{c}c)(\bar{d}u), [\bar{c}\bar{d}][cu]$$

- Horizontal lines: energies of expected two-meson states in limit of no interaction: $E = E[M_1(p_1)] + E[M_2(p_2)]$
- Circles: energies of eigenstates from latt
- Only expected two-meson states observed.
- No lattice candidate for charged X(3872). In agreement with absence of such state in exp.
- No lattice candidate for other charged state. Two Belle 2008 states are exp. unconfirmed.

[M. Padmanath, C.B. Lang, S.P., 1503.03257]

$\Upsilon(4140)$, $J^{PC}=?^?+?$, ccss

Experiment:

peak in $J/\psi \Phi$ just above $J/\psi \Phi$ threshold

found: CDF 2009, CMS 2012, D0 2013, Babar 2015

not found: Belle 2010, LHCb 2012

Lattice:

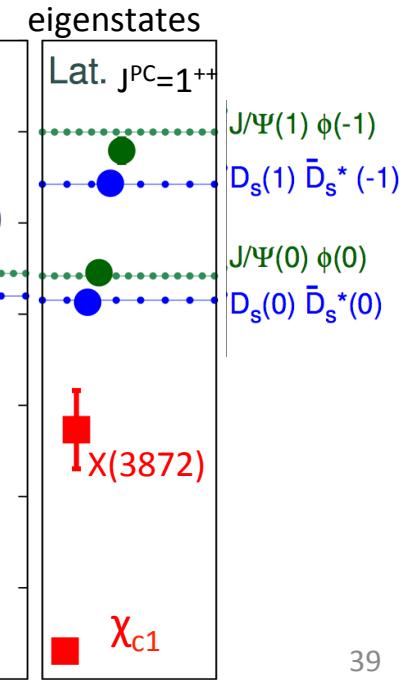
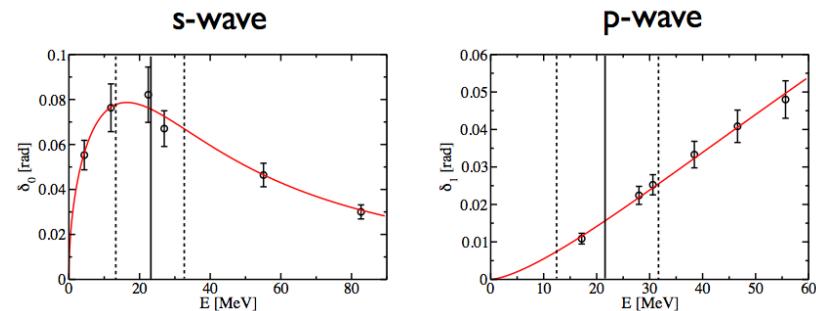
- S. Ozaki and S. Sasaki, 1211.5512, PRD
strange quark annihilation neglected
no resonant structure found for
- M. Padmanath, C.B. Lang, S.P., 1503.03257
 $\mathcal{O} : \bar{c}c, (\bar{c}s)(\bar{s}c), (\bar{c}c)(\bar{s}s), [\bar{c}\bar{s}][cs]$

channel $J^P=1^+$ considered only: expected two-particle eigenstates found and χ_{c1} , $X(3872)$ but not $\Upsilon(4140)$

$$Y(4140) \rightarrow J/\psi \phi$$

$$\bar{c}c \quad \bar{s}s$$

$J/\psi \Phi$ scattering phase shift [rad]



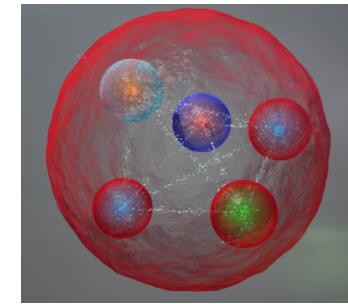
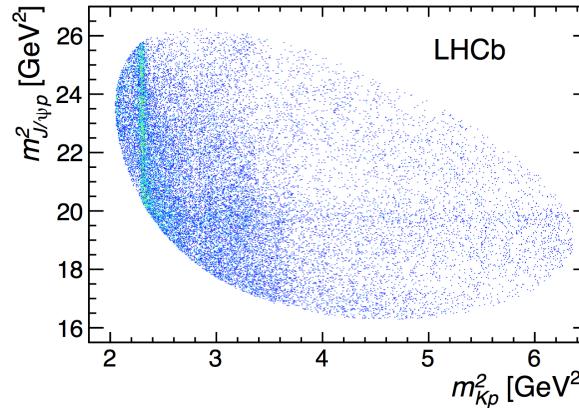
Conclusions

- ◆ **Experiment** (in brief): exciting tetraquark and pentaquark candidates
- ◆ **Phenomenology**: many many interpretations of exotic experimental “bumps”, some of them do not need to introduce exotic hadrons to explain experimental data
- ◆ **Lattice QCD** (in brief):
 - Evidence found for many hadrons with non-exotic flavor:
 - states well below th. : charmonium , D, π , K ... and all the others
 - shallow bound states : D_{s0} , D_{s1} , B_{s0} , B_{s1} , X(3872) with I=0
 - resonances via BW : ρ , K^* , $K_0^*(1430)$, K_2 , D_0^* , D_1 , a_1 , b_1 , $\Psi(3770)$
 - Hadrons with exotic flavor
 - if exotic hadrons were below strong decay threshold, they would be easy to (dis)prove in LQCD
 - unfortunately, most of exotic hadrons can decay into several channels via strong interaction
 - therefore lattice has not given yet a final answer which (if any) exotic hadrons arise from QCD
 - I have (hopefully) given you some flavor in which direction progress is going
 - this is an exciting topic at present and I am looking forward to face further challenges it poses ...

Backup slides

Pentaquarks

$$\Lambda_b^- \rightarrow K^- J/\psi p$$



LHCb: 1507.03414

14th July 2015

$$P_c^+ \rightarrow J/\psi \quad p$$

$$\bar{c}c \quad uud$$

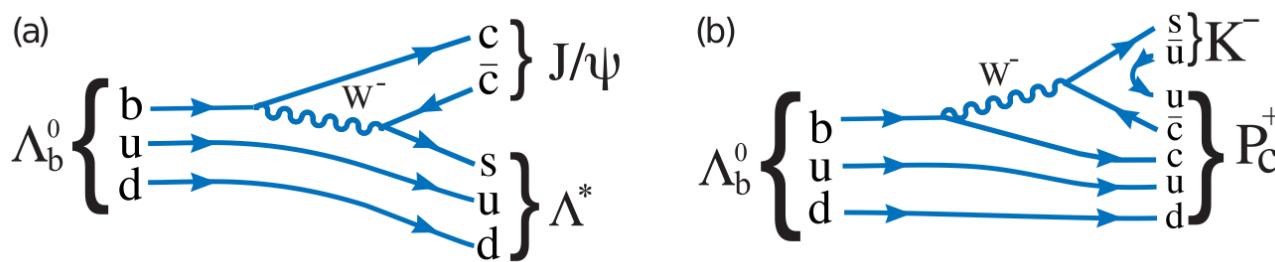
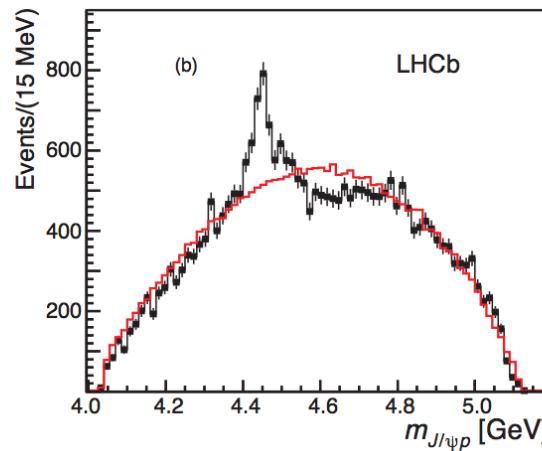
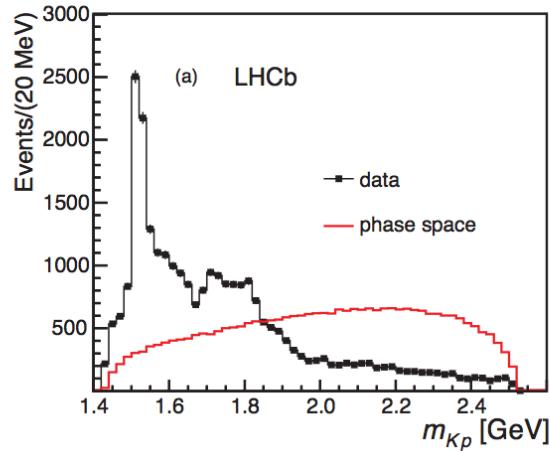


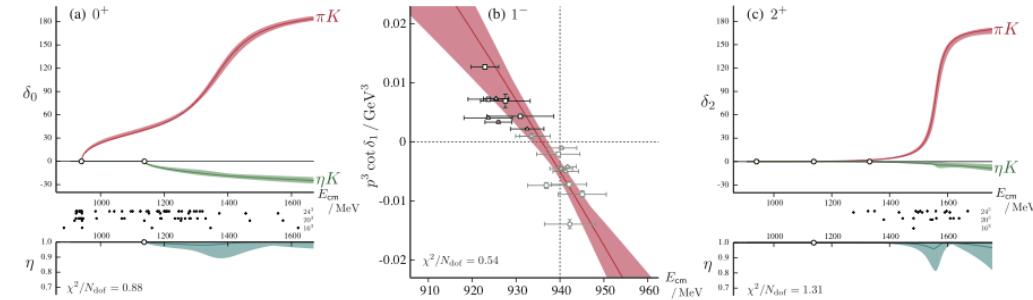
Figure 1: Feynman diagrams for (a) $\Lambda_b^0 \rightarrow J/\psi \Lambda^*$ and (b) $\Lambda_b^0 \rightarrow P_c^+ K^-$ decay.



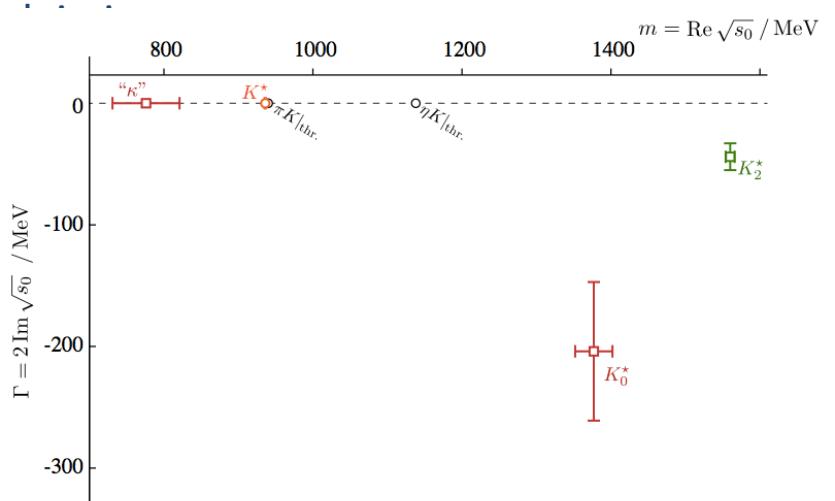
State	J^P	M_0 (MeV)	Γ_0 (MeV)
$\Lambda(1405)$	$1/2^-$	$1405.1^{+1.3}_{-1.0}$	50.5 ± 2.0
$\Lambda(1520)$	$3/2^-$	1519.5 ± 1.0	15.6 ± 1.0
$\Lambda(1600)$	$1/2^+$	1600	150
$\Lambda(1670)$	$1/2^-$	1670	35
$\Lambda(1690)$	$3/2^-$	1690	60
$\Lambda(1800)$	$1/2^-$	1800	300
$\Lambda(1810)$	$1/2^+$	1810	150
$\Lambda(1820)$	$5/2^+$	1820	80
$\Lambda(1830)$	$5/2^-$	1830	95
$\Lambda(1890)$	$3/2^+$	1890	100
$\Lambda(2100)$	$7/2^-$	2100	200
$\Lambda(2110)$	$5/2^+$	2110	200
$\Lambda(2350)$	$9/2^+$	2350	150
$\Lambda(2585)$?	≈ 2585	200

Resonances in $K\pi$, $K\eta$ coupled channels

- qq , $K\pi$, $K\eta$ interpolators
- a number of different $0 < P \leq 2$
- for each E_n : one determinant equation for many unknowns
- T-matrix parametrized to get around this problem
- the location of poles of T-matrix in complex given below
- $K^*(892)$ and κ are below threshold for this m
- K_0^* , K_2^* are resonances
- $m_\pi = 391 \text{ MeV}$, $N_L = 16, 20, 24$
[*Dudek, Edwards, Thomas, Wilson, HSC, 1406.4158, PRL 2014*]



$$t_{ii} = \frac{(\eta e^{2i\delta_i} - 1)}{2i\rho_i}, t_{ij} = \frac{\sqrt{1-\eta^2} e^{i(\delta_i + \delta_j)}}{2\sqrt{\rho_i \rho_j}}$$

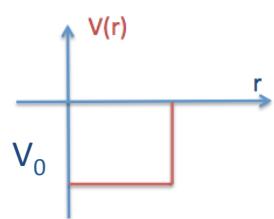


$$\det \left[\delta_{ij} \delta_{JJ'} + i\rho_i t_{ij}^{(J)}(E_{\text{cm}}) \left(\delta_{JJ'} + i\mathcal{M}_{JJ'}^{\vec{P}\Lambda}(p_i L) \right) \right] = 0,$$

location of poles in T matrix in complex plane

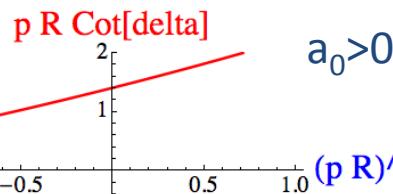
How to identify a shallow bound state? (continued)

V_0 that provides no bound st.

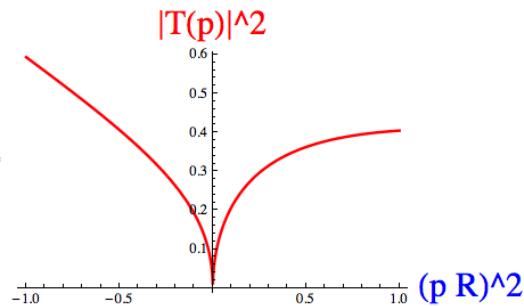


$$\lim_{p \rightarrow 0} p \cot \delta(p) = \frac{1}{a_0}$$

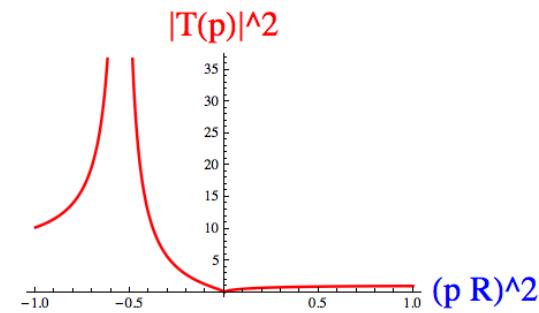
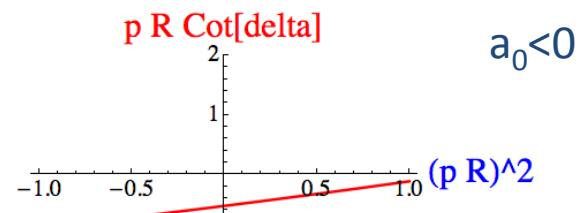
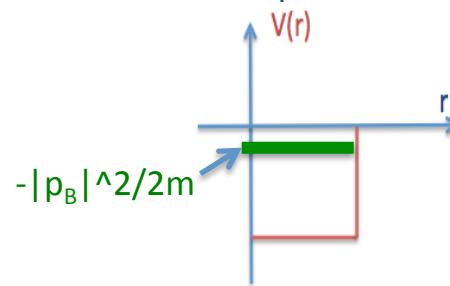
a_0 =scattering length



$$|T(p)|^2 \propto \left| \frac{1}{\cot \delta - i} \right|^2$$



V_0 that provides one shallow bound st.

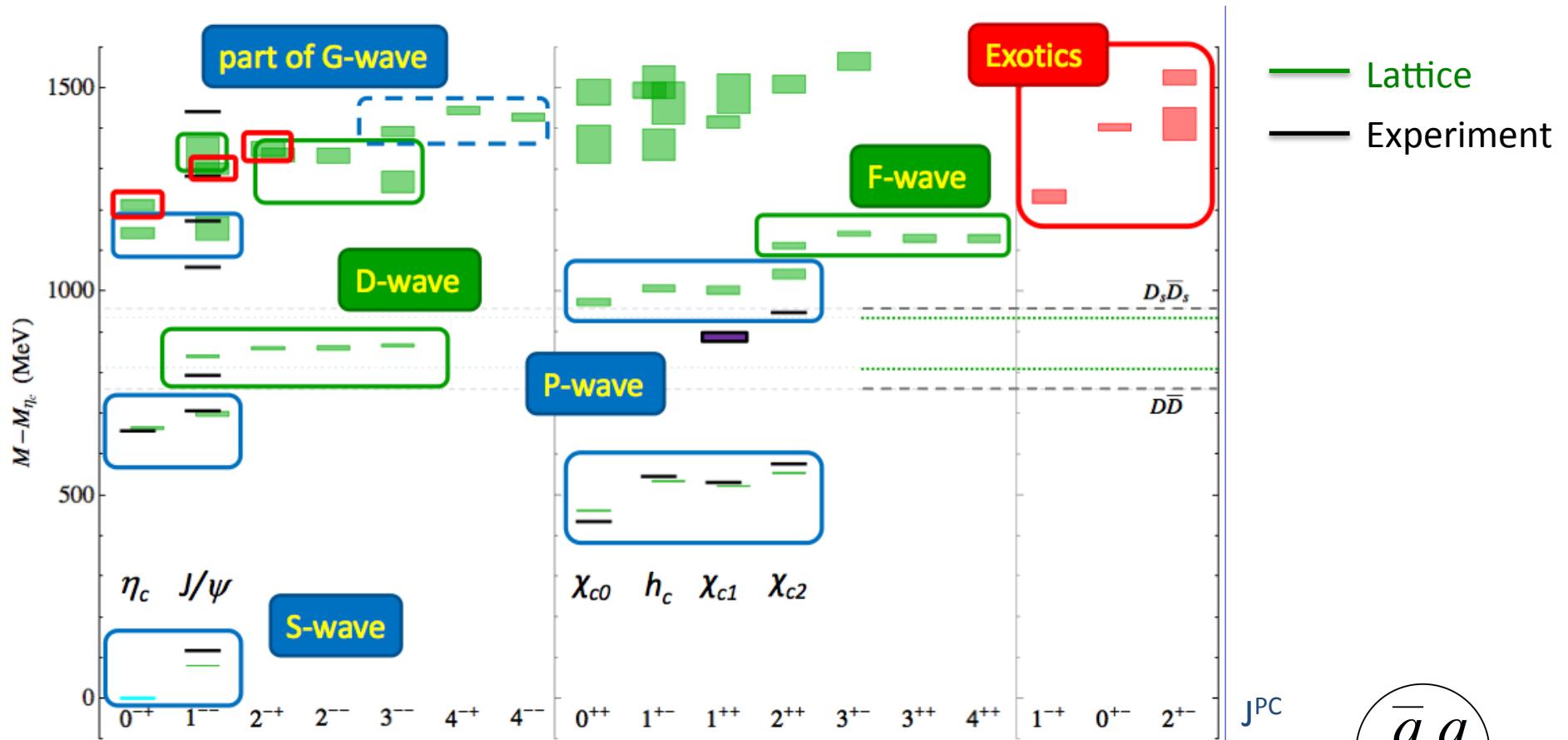


$-(|p_B| R)^2$
scattering matrix
has a pole

Hadrons near or above threshold: single-hadron approximation

- only interpolating fields $\mathcal{O} \approx \bar{q} q$ or $\mathcal{O} \approx q q q$
 - assumptions: all energy levels correspond to "one-hadron" states
no two-hadron state is seen
- $m=E$ (for $P=0$)
these are strong assumptions ...
but results still present valuable reference point

cc spectrum: single-hadron approximation



[HSC , L. Liu et al: 1204.5425, JHEP]

- $m_\pi \approx 400$ MeV, $L \approx 2.9$ fm, $N_f = 2+1$
- identification with $n^{2S+1}L_J$ multiplets using $\langle O | n \rangle$
- green: lat, black: exp

Hybrids:

some of them have exotic J^{PC}
large overlap with $O = \underline{q} F_{ij} q$

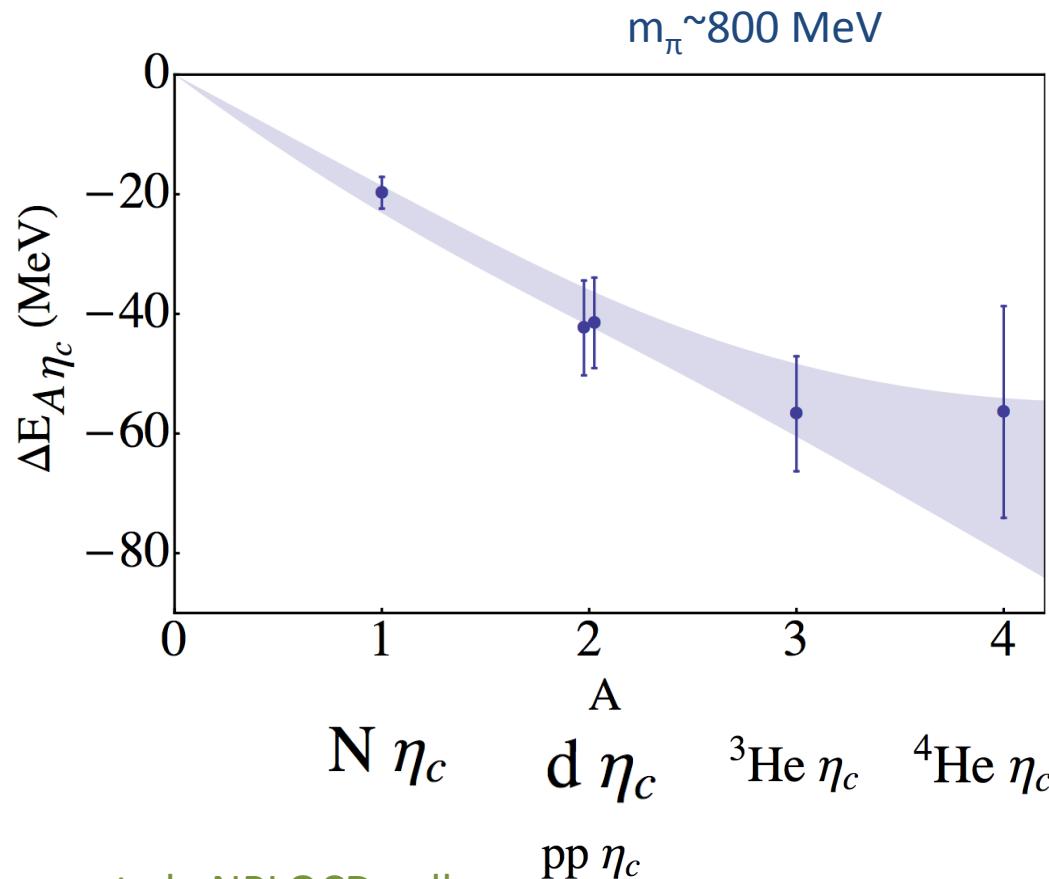


Bound states of a charmonium and a nucleus

Exp: no reliable candidates for such states yet

Lat: SU(3) symmetric point, $m_u=m_d=m_s$, $m_\pi \sim 800$ MeV, $L=3.4, 4.5, 6.7$ fm

Bound states found at this m_π . Not known yet whether these bound states survive at m_π^{phys}



[Beane et al., NPLQCD coll. ,

1410.7069]

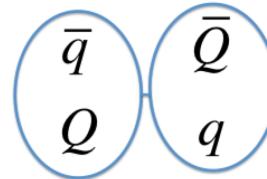
Binding energies
for a system composed of
 $\eta_c + \text{nucleus } (A)$

number of nucleons
in nucleus

Phenomenological approaches

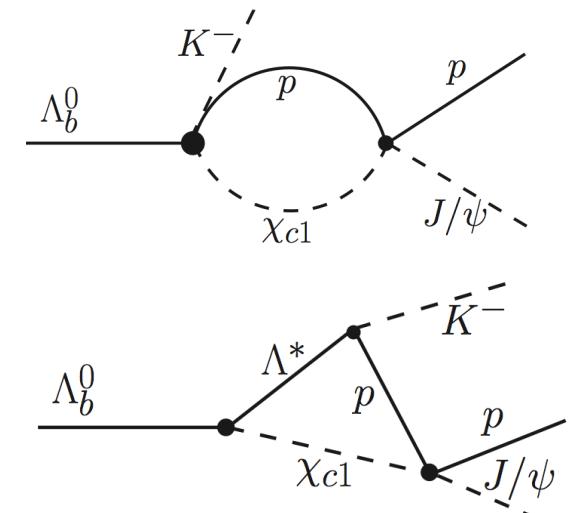
Approaches that support new exotic hadrons

- compact tetraquark: diquark antiquark
- meson molecules
- ...



Approaches that do not contain new exotic hadrons, and explain bumps in experimental cross sections via

- kinematical effects due to nearby threshold
- coupled channel effects
- kinematics interplay in triangle diagrams
- ...



[Guo, Meissner, Wang, Yang
1507.04950]

I will try to address the question whether observed exotic and conventional hadrons arise directly from QCD.

ρ resonance: lattice results

from T. Yamazaki [lat14 plenary, 1503.0867]

$$\Gamma = \frac{g_{\rho\pi\pi}^2}{6\pi} \frac{p^3}{E^2}$$

